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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

VISCOSITY AND SURFACE TENSION EFFECTS ON V-NOTCH WEIR COEFFICIENTS

By Arno T. Lenz,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Results of 652 test runs in the Hydraulic and Sanitary Laboratory at the University of Wisconsin, in Madison, are reported in this paper. Square-edged, brass, V-notch weirs, with six different angles, were tested, with heads up to 0.88 ft, using water and two oils at several temperatures.

Viscosity varied from 1 to 150 times that of water; surface tension from 1 to 0.41 times that of water; and density from 1 to 0.85 times that of water. Also reported are 45 tests made at the University of California, in Berkeley, with two oils, at heads up to 0.20 ft, and 92 tests made at Cornell University, in Ithaca, N. Y., with water, at heads up to 3.43 ft.

The purpose of the Wisconsin tests was to obtain experimental data with regard to the effect of viscosity and surface tension on a weir coefficient C. It was found that the weir coefficient increases with both viscosity and surface tension, and that it decreases with increase in head and angle of V-notch.

A general equation expressing these relations mathematically was derived with the aid of dimensional analysis and is given in the paper, along with the limiting conditions beyond which the equation is not valid. As it should be good for any oil, its validity was checked with previously unpublished data from tests made at the University of California with two different oils. The coefficients computed by the general equation were within 1% of the test values.

Comparison of published coefficients, from similar tests with water, with the values of C computed from the general equation shows agreement within about 1% for all weir angles between 28° and 90°. Head-discharge equations are given which express the results of these water tests.

Notation

The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference in the Appendix.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by July, 1942.

[1] Madison, Wis.

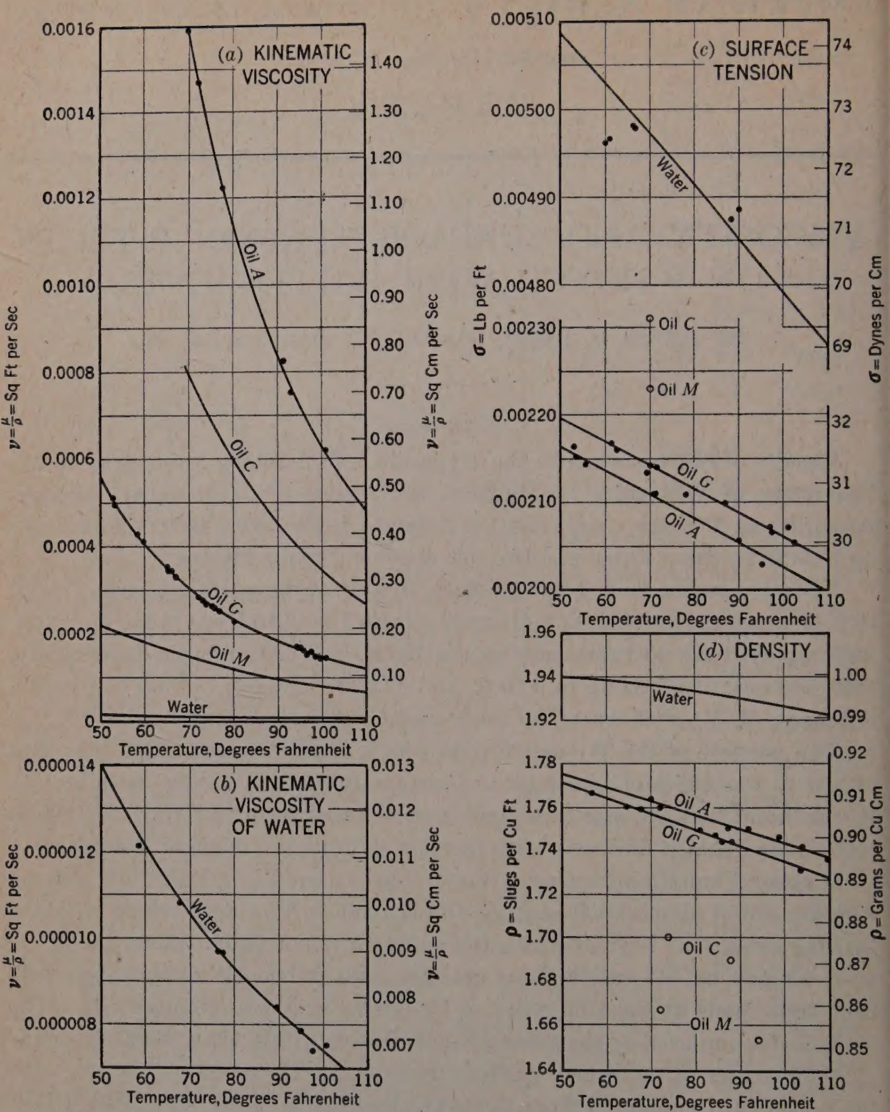


FIG. 1.—PHYSICAL PROPERTIES OF LIQUIDS TESTED

INTRODUCTION

Since the first work on V-notch (triangular) weirs by Prof. James Thomson (13)[2] in 1858, many experimenters starting with the equation

$$Q = c\, H^{2.5} \qquad (1)$$

have determined the value of the coefficient c for various values of head, angle of notch, type of weir crest, size of approach channel, and temperature of

[2] Numerals in parentheses, thus: (13), refer to corresponding items in the Bibliography (see Appendix).

flowing liquid. The effect of the angle of notch is largely eliminated by the use of the formula,

$$Q = C \frac{8}{15} \sqrt{2\,g} \tan \frac{\theta}{2} H^{2.5} \qquad (2)$$

and since this is the most general fundamental equation available it is used as the starting point in this paper. In Eqs. 1 and 2, Q, of course, is the flow; c is an empirical coefficient; H is the hydraulic head; g is the gravity constant; and θ is the angle of the notch. The value C in Eq. 2 is really the dimensionless ratio of the actual discharge to the discharge as computed by calculus methods, usually termed the theoretical discharge. Therefore, C should express the effect of the various physical properties of the flowing liquid, other factors being constant. The methods of dimensional analysis have been applied to this problem by Hubert Mawson (8), B. M. Thornton, Ed S. Smith, Jr. (11), and H. N. Eaton (5), but all the experimental work by which they prove their conclusions was done with only one liquid—water. Since the physical properties of water change little with temperature, it was necessary to experiment with liquids having widely different properties in order to show definitely how the value of C depends on viscosity, surface tension, and density.

Three groups of new test data are reported. They are: (*a*) Tests made by the writer and assistants at the University of Wisconsin, (*b*) tests made at the University of California under the direction of Morrough P. O'Brien, M. Am. Soc. C. E., and (*c*) tests made by Chitty Ho, Assoc. M. Am. Soc. C. E., and Sze-Ling Wu at Cornell University (7). The trade names of the various oils used have been replaced by arbitrary letter designations, serving the same purpose as far as discussion is concerned. Their physical characteristics are given in Fig. 1.

Experimental Methods

Wisconsin Tests.—The apparatus used in all tests with water and oil G is shown in Fig. 2. The weir tank had for its front end a frame of 2-in. steel angles. Two steel end plates, one above the other, closed the opening in the end frame. A small opening through which the liquid could flow was cut in the upper plate to which were bolted two brass weir plates as shown in Figs. 3 and 4. Two designs of upper end plate were used—one for the 10°, 20°, 28°, and 45°, and one for the 60° and 90°, V-notches. It was thus possible to use the same two weir plates for tests at all angles.

The weir plates were $\frac{1}{8}$-in. brass plates, $8\frac{1}{2}$ in. by 13 in., beveled to $\frac{1}{32}$-in. thickness at the weir crest, with square upstream corners and straight edges. A filler plate with the correct angle was in place below the weir plates, as shown in Fig. 3, throughout each series of runs, in order to provide a smooth upstream surface of at least $8\frac{1}{2}$ in. from the crest in all directions.

The point of the notch was 3 ft above the bottom of the tank. This gave a ratio of approach-channel depth to head of 4.4 for the maximum head of 0.88 ft with the 10° notch and 7.0 for the maximum head of 0.50 ft with the 90° notch. The width of the channel of approach (3.5 ft) was 7.0 times the head for the maximum head on the 90° notch. The work of James Barr (2) would

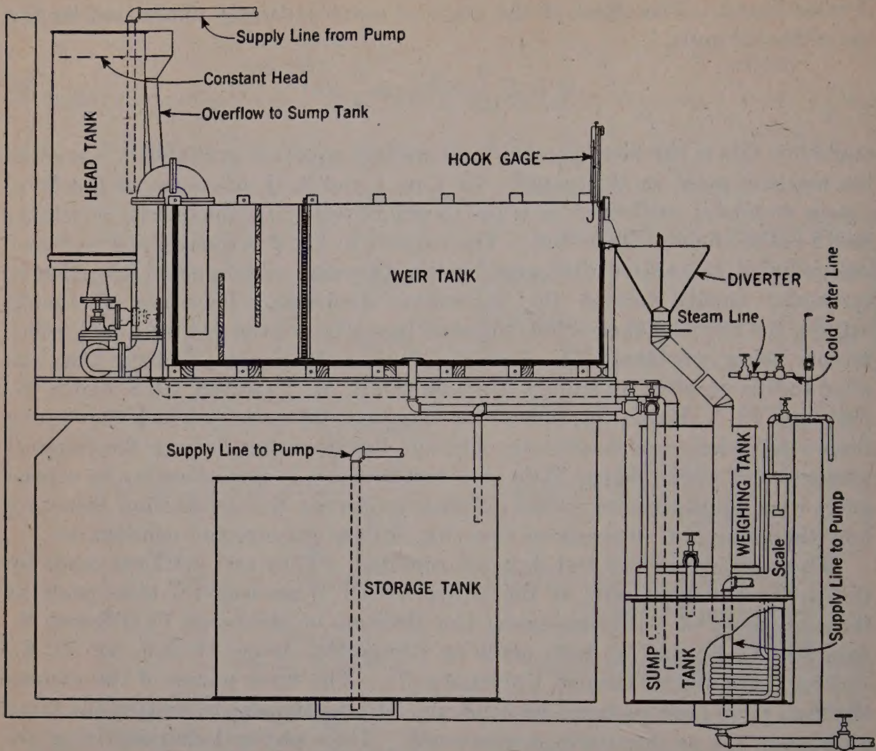


FIG. 2.—WEIR APPARATUS (SECTIONAL ELEVATION)

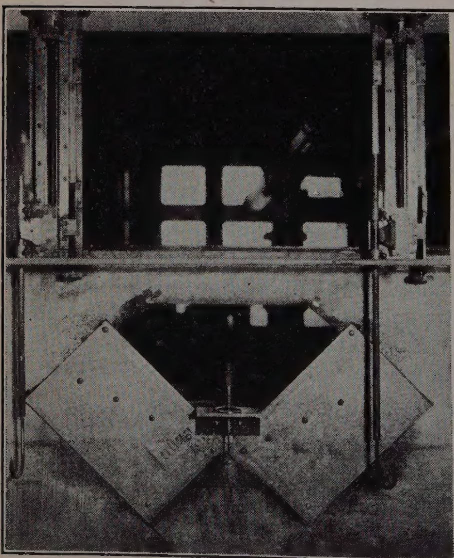


FIG. 3.—APPARATUS FOR DETERMINING ZERO READING

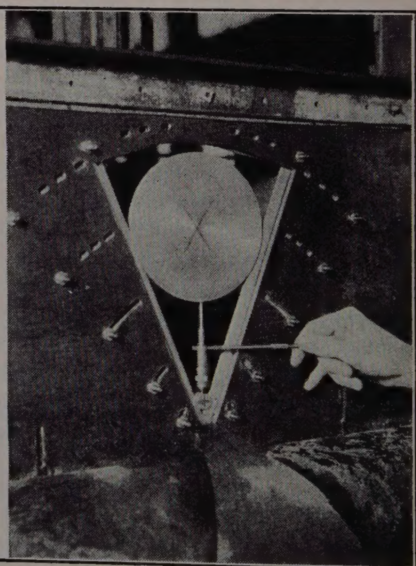


FIG. 4.—MEASUREMENT OF ANGLE OF NOTCH

indicate that there should be no influence due to side-walls or floor with ratios of this magnitude.

The liquid flowed over the notch into a funnel-shaped diverter (Fig. 2) from which it could be discharged into the sump tank or into either one of two weighing tanks. A coil in the sump tank was provided with steam and cold-water connections and was used to maintain a constant temperature in the liquid. From the sump tank the liquid was pumped to a constant-head tank from which it flowed to the weir tank. The liquid entered the weir tank behind a series of three baffles, the last of which was 7 ft 9 in. from the weir plates. The flow from the last baffle to the weir plates was exceedingly smooth, and the head was maintained constant throughout each run.

The apparatus used in the tests with oil A was essentially the same as with oil G, with one exception. Each V-notch was cut in a single brass plate 12 in. by 24 in. by $\frac{3}{32}$ in. thick. The edges were beveled to the same profile as in the other tests. The principal objection to this type of notch was the difficulty in fixing the exact vertex of the V-notch at the geometric intersection of the sides, particularly for the narrow angles.

Measurements.—The head was measured by two hook gages reading to 0.0001 ft, one on each side of the notch and rigidly fastened to the headframe as shown in Fig. 3. A third gage 5 ft upstream from the weir was also read until 300 runs proved beyond doubt that the gages fastened to the headframe were beyond any drawdown influence from the notch. This method of reading heads was much more satisfactory than the use of a hook gage in a stilling well, particularly at high temperatures, for it was found that the level of the quiet liquid in the well was not the same as in the weir tank, the difference in level being in proportion to the difference in liquid density due to the temperature variation between stilling well and weir tank.

Zero readings on the hook gages were obtained by measuring with a small auxiliary micrometer hook gage from a shelf resting on a small disk placed in the V-notch, to the level of the quiet liquid in the tank (Fig. 3), and computing the difference in level between the liquid and geometrical point of the notch. The angle of notch was obtained by measuring with a micrometer caliper between two disks of known diameter and computing sin $\theta/2$ (Fig. 4). For the 90° notch a level bar was used instead of the upper disk.

The actual discharge over the weir was determined by weighing the liquid caught in a measured time. For small rates of flow a small scale, accurately calibrated, measured the weight to 0.01 lb. For a net weight of more than 300 lb a large tank with a maximum capacity of 1,500 lb was used. The scale was sensitive to 0.2 lb. Runs of 5-min duration were made for low rates of flow, the time decreasing to about 1 min for high rates. Time was measured by an accurate stop-watch, graduated to 0.2 sec, with the reading estimated to 0.1 sec.

Temperature was read on a thermometer graduated to half degrees Fahrenheit, and readings were estimated to the nearest 0.1°. The correct unit weight for the actual temperature of the liquid was used in computing the volumetric rate of discharge. The value of g was taken as 32.169 ft per sec^2 (980. 5cm per sec^2). It is believed that all measurements taken are accurate within 0.2%, most of them being accurate to within 0.1%.

Physical Properties of Liquids.—Oil A was a fuel oil, No. 5 grade, the type used in large oil burners. Oil G was a dustproofing oil, the type used to spray on coal to reduce dust trouble before delivery. Both were selected because of their viscosity range and also because the 2,200 gal required in the circulating system could be resold after the tests were completed. Lake water was used in all the water tests. The appearance of the liquid jet for oil G may be seen in Fig. 5.

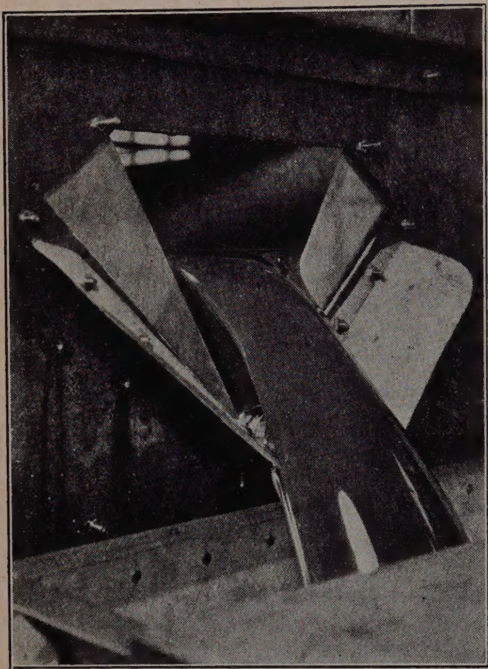


FIG. 5.—OIL G FLOWING OVER A 60° V-NOTCH WEIR

The density (ρ) of the liquids used in the tests is shown in Fig. 1(*d*). The values for oils A and G were obtained by tests with the hydrometer and Westphal balance, and by weight determinations, using flasks of known volume. Plotted points in Fig. 1(*d*) indicate test results.

Kinematic viscosity (ν) was measured by means of an Ostwald viscosimeter. The instrument was calibrated with distilled water as recommended by the American Society for Testing Materials (A.S.T.M.) Standards (1). The curve in Fig. 1(*b*) is from the International Critical Tables (9), whereas the plotted points were obtained from water measurements using the instrument constant obtained by calibration. Test data for oils and a comparison with the water curve is shown in Fig. 1(*a*). Computations for Reynolds' number **R** were made using values taken from the smooth curves.

Surface tension (σ) was determined by measuring the rise in the capillary tube of the apparatus shown in Fig. 6. The difference in level between the liquid in the two legs of the U-tube is directly proportional to the surface tension of the liquid and inversely proportional to ρ, g, and an instrument constant. This capillary rise was measured with a micrometer comparator telescope reading to 0.005 mm. Comparison of the test data obtained with distilled water with the values published in the International Critical Tables (9) is given in Fig. 1(*c*). Test data from oil measurements and the mean curves used in computations are also shown. The water bath shown in Fig. 6 was used for both the surface tension and viscosity measurements to hold the temperature constant.

California Tests.—The two oils M and C were used in the California tests, which were conducted at low heads (all less than 0.20 ft) and with 90° weirs. Tests were made with each oil at two temperatures, approximately 70° F and

90° F. The experimental data are summarized in Table 1(*a*). The tank used consisted of a weir box 14½ in. high, 10⅜ in. wide, and 40¾ in. long. It contained two baffles, the last one 24¼ in. upstream from the 3¼-in. by 6-in. brass weir plate in which was cut a 90° V-notch. A head gage in a stilling well 3½ in.

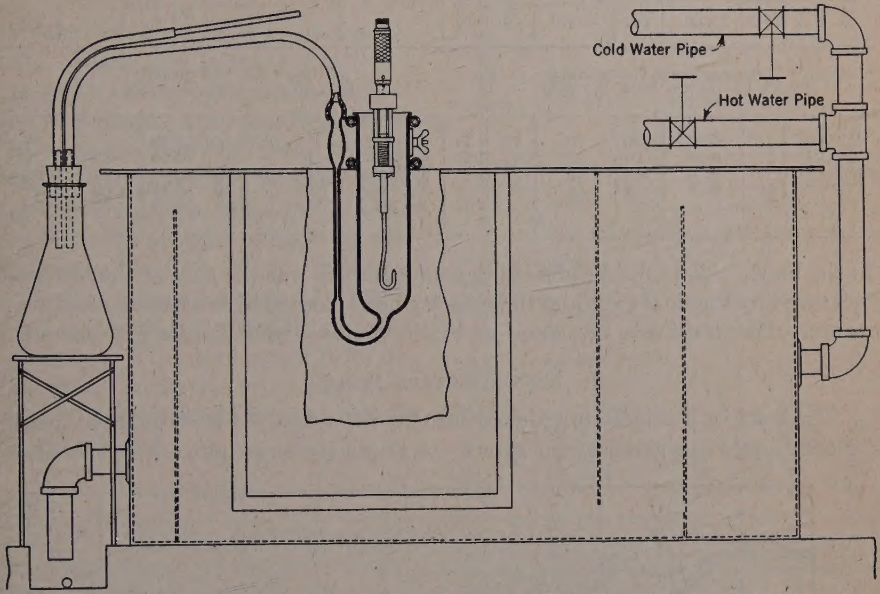


FIG. 6.—ARRANGEMENT OF SURFACE TENSION U-TUBE IN A CONSTANT-TEMPERATURE BATH

in diameter was connected to the weir tank with a ⅜-in. tube, the connection being 15¾ in. upstream from the weir plate. Maximum head in any of the test runs was 0.20 ft; minimum head selected from the data for use in this paper was 0.10 ft. Head-gage readings were recorded to 0.001 ft; time to 0.1 sec, with a minimum of 15 sec at 0.20-ft head; and weights to 0.1 lb, with a minimum net weight of 30 lb.

Cornell Tests.—All the Cornell tests were made with water only, but with five angles of notch, 90°, 60°, 37°, 28°, and 19°, at heads up to 3.43 ft (see Table 1(*b*). They were made with "brass plates ⅛ in. thick and 6 in. wide with sharp square edges in order to make the water sheet discharge freely." These plates were attached to a bulkhead near the end of the same concrete channel—6 ft wide and 11.6 ft deep—that was used in the weir experiments by E. W. Schoder, M. Am. Soc. C. E., and the late K. B. Turner (10), first published in 1927. The vertex of each notch was set 8 ft above the floor level. The closest baffle was 24.6 ft from the weir. Heads were measured with float gages connected 11.74 ft upstream from the notch. One float gage was equipped with a vernier, and readings were recorded to 0.0001 ft; the other was read to 0.01 ft and was used as a check only. Velocity measurements upstream from the weir were made either before or after the runs at relatively high heads by means of wooden floats, the submerged depth of the float being adjusted equal

TABLE 1.—Summary of Experimental Data

Series	Oil	Runs (45)	Range of Head (Feet) From:	Range of Head (Feet) To:	Range of Temperature (Degrees F) From:	Range of Temperature (Degrees F) To:	Series	Exact angle of notch	Runs (92)	Range of Head (Feet) From:	Range of Head (Feet) To:	Range of Temperature (Degrees F) From:	Range of Temperature (Degrees F) To:
(a) University of California (Angle of Notch Exactly 90°)							(b) Cornell University (All Tests with Water)						
AC	M	12	0.12	0.20	70	73	D	90° 0′	15	0.15	2.19	..	67
AD	M	12	0.12	0.20	92	95	E	60° 0′	17	0.15	2.98	68	70
DC	C	11	0.10	0.18	69	75	F	36° 52′.2	18	0.32	3.43	71	73
DD	C	10	0.12	0.18	88	..	G	28° 4′.4	19	0.29	3.17	68	..
..	..	..	....	...	..	..	H	18° 55′.5	23	0.18	3.37	72	73

to the head. The total head used in computations was the sum of the head as measured by the float gage and the velocity head computed from these measurements. Discharge was measured as in the Schoder and Turner experiments.

Experimental Data

The data in Table 2 are grouped into 20 series, one for each angle of notch for each liquid at a given temperature. A duplicate series number indicates an

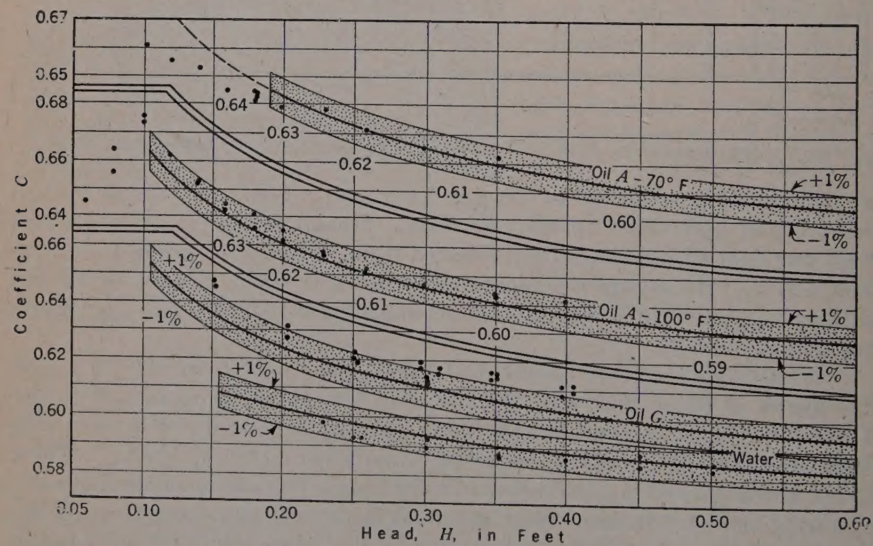


Fig. 7.—Head-Coefficient Curves for a 90° V-Notch

entirely independent set of runs. The original data are on file at the University of Wisconsin, and a summary of data for each run is on file in the Engineering Societies Library.[3]

Head-Coefficient Data and Curves.—Experimental coefficients are listed in Tables 1 and 2, and the data for oils A and G are shown in Fig. 7. The

[3] 29 West 39th Street, New York, N. Y.

curves in Fig. 7 are computed from the formula

$$C = 0.56 + \frac{B}{\mathbf{R}^n\,\mathbf{W}^m} \qquad (3)$$

in which B is a known constant; m and n are constant exponents; and $\mathbf{W}$ = Weber's number. All test points within the shaded band are within 1% of the corresponding computed value. The broken line at low heads (oil A at 70° F) indicates the extension of Eq. 3 beyond its range of usefulness. At high heads the coefficient tends to become constant. Similar curves plotted for all the data in Tables 1 and 2 showed that nearly all test points are within 1% of the corresponding values computed by Eq. 3, and that within the useful range the variation of the test points from the equation seems to be unrelated to head, viscosity, surface tension, or angle of notch.

TABLE 2.—SUMMARY OF EXPERIMENTAL DATA, WISCONSIN TESTS

Series[a]	Angle of notch	Runs[b]	Range of Head (Feet) From:	Range of Head (Feet) To:	Range of Temperature (Degrees F) From:	Range of Temperature (Degrees F) To:
90A 70	89° 53′.9	13	0.10	0.35	69.8	70.2
90A 100	89° 53′.9	14	0.06	0.35	99.8	100.2
90A 100	89° 53′.9	13	0.06	0.40	99.9	100.3
90G	89° 49′.6	23	0.15	0.40	64	68
90D	89° 53′.2	14	0.23	0.50	61	63
60G	59° 56′.9	23	0.15	0.55	62	67
60D	59° 51′.0	33	0.16	0.60	54	62
45A 70	44° 52′.0	16	0.10	0.50	69.7	70.4
45A 70	44° 52′.0	27	0.18	0.50	69.6	72.2
45A 100	44° 52′.0	25	0.10	0.55	99.2	100.1
45A 100	44° 52′.0	32	0.10	0.55	99.5	102.0
45G	44° 40′.0	22	0.18	0.60	58	65
45D	44° 40′.0	18	0.20	0.55	56	60
28A 70	28° 13′.1	28	0.16	0.55	70.1	70.4
28A 70	28° 13′.1	35	0.16	0.60	71.5	74.0
28A 100	28° 13′.1	24	0.10	0.55	99.6	100.0
28A 100	28° 13′.1	36	0.10	0.60	98.9	100.6
28G	27° 53′.0	25	0.18	0.70	64	69
28D	27° 55′.0	18	0.26	0.70	51	59
20A 70	19° 54′.7	27	0.14	0.60	70.1	70.3
20A 70	19° 54′.7	9	0.18	0.55	70.0	71.7
20A 100	19° 54′.7	33	0.10	0.60	99.3	101.0
20A 100	19° 54′.7	14	0.10	0.55	99.7	100.1
20G	19° 56′.3	26	0.18	0.70	64	69
20D	19° 55′.0	24	0.18	0.70	54	67
20D	20° 2′.5	19	0.24	0.70	54	65
10G	10° 7′.8	28	0.20	0.80	58	64
10D	10° 20′.1	33	0.24	0.88	52	54

[a] "A" denotes "oil A"; "G" denotes "oil G"; and "D" denotes "water." [b] Total, 652.

Theory

The derivation of a general equation that should be applicable to any liquid flowing over a V-notch weir is aided by a dimensional analysis of the problem. Although the equation derived (Eq. 3) is an empirical one, the fact that it is dimensionally correct should remove some objections to its use for liquids other than those actually tested.

The discharge Q over a V-notch weir is a function of the measurements shown in Table 3. As b and A are functions of the angle θ, there remain, for a notch of given angle, six independent variables. From dimensional analysis it is known that

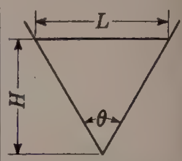
$$\phi\,(V, H, \rho, \mu, \sigma, g) = 0 \qquad (4a)$$

According to the π-theorem (3a), a complete equation of the form of Eq. 4a has the solution:

$$\phi'\,(\pi_1, \pi_2, \cdots) = 0 \qquad (4b)$$

in which π_1, π_2, π_3, etc., are the independent products of the arguments V, H, ρ, $\cdots$, and are dimensionless in the fundamental dimensional units: Force (F), length (L), and time (t). In this case there are three terms in the π-theorem, each a dimensionless product of powers of the six independent variables.

TABLE 3.—VARIABLE QUANTITIES IN THE MEASUREMENT OF FLOW

No.	Measurement	Symbol	Unit	Dimensions[a]	Weir section
1	Head	H	Feet	L	
2	Crest length (projected)	L	Feet	....	
3	Angle of notch (projected)	θ	Degrees	....	
4	Area of notch (projected)	A	Square feet	....	
5	Mean velocity $= Q/A$	V	Feet per second	L/t	
6	Density	ρ	Slugs per cubic feet	$F\,t^2/L^4$	
7	Viscosity	μ	Pound-seconds per square foot	$F\,t/L^2$	
8	Surface tension	σ	Pounds per foot	F/L	
9	Acceleration of gravity	g	Feet per second per second	L/t^2	

[a] Six independent variables.

Since $\frac{\mu}{\rho} = \nu$, Eq. 4*b* thus becomes

$$\phi''\left(\frac{\nu}{V\,H}, \frac{\sigma}{V^2\,\rho\,H}, \frac{g\,H}{V^2}\right) = 0 \quad \ldots\ldots (5)$$

For a given angle of V-notch it is then possible to write:

$$V = \sqrt{g\,H}\,f\left(\frac{\nu}{V\,H}, \frac{\sigma}{V^2\,\rho\,H}\right) \quad \ldots\ldots (6a)$$

or

$$Q = A\,V = \sqrt{g}\tan\frac{\theta}{2}\,H^{2.5}\,f\left(\frac{\nu}{V\,H}, \frac{\sigma}{V^2\,\rho\,H}\right) \quad \ldots\ldots (6b)$$

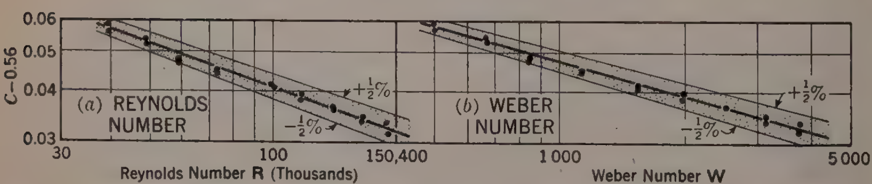


FIG. 8.—COEFFICIENT CURVES FOR WATER (56° TO 62° F) FLOWING THROUGH A 45° V-NOTCH WEIR

When Eq. 6*b* is compared with Eq. 2, a solution is obtained for the weir coefficient C:

$$C = f'\left(\frac{\nu}{V\,H}, \frac{\sigma}{V^2\,\rho\,H}\right) \quad \ldots\ldots (7)$$

The value of C for a given angle of notch, therefore, is a function of the kinematic viscosity and surface tension of the flowing liquid. Since C is to be used to compute the mean velocity of flow and the actual discharge, the presence of the factor V in each of the foregoing dimensionless numbers is a serious disadvantage even if the functional relationship between C and the dimensionless products were known. It is proposed, therefore, to replace V in each of the products by a dimensional equivalent, $\sqrt{g\,H}$. Then

$$C = f''\left(\frac{\nu}{g^{0.5}\,H^{1.5}}, \frac{\sigma}{\rho\,g\,H^2}\right) \quad \ldots\ldots (8)$$

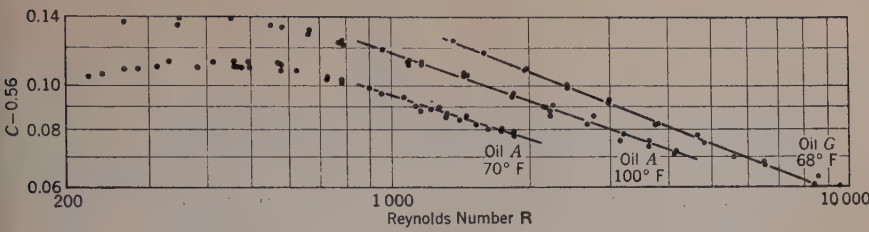


FIG. 9.—COEFFICIENT CURVES FOR OIL FLOWING THROUGH A 28° V-NOTCH WEIR

Taking the reciprocal value in each case to obtain whole numbers, let Reynolds' number

$$\mathbf{R} = \frac{g^{0.5}\, H^{1.5}}{\nu} \qquad (9a)$$

and Weber's number

$$\mathbf{W} = \frac{\rho\, g\, H^2}{\sigma} \qquad (9b)$$

because they are dimensionless numbers expressing the effect of viscosity and

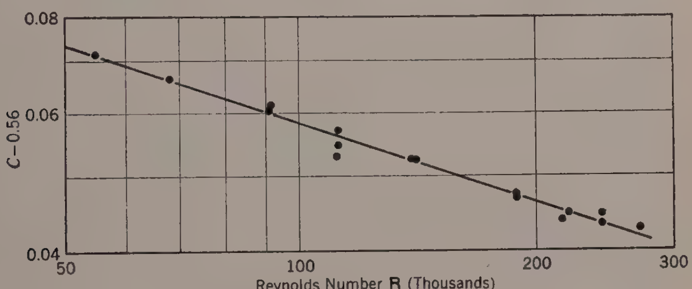


FIG. 10.—COEFFICIENT CURVE FOR WATER AT 56° F, FLOWING THROUGH A 28° V-NOTCH WEIR

surface tension under the given flow conditions. However, the exact relationship between V and H is not expressed, and therefore the usual critical values of these numbers for pipe flow cannot be applied to V-notch weir flows. Then

$$C = f''\left(\frac{1}{\mathbf{R}}, \frac{1}{\mathbf{W}}\right) \qquad (10)$$

ANALYSIS OF DATA

Development of General Equation.—The functional relationship, f'', between C, **R**, and **W** for a given weir must be obtained from experimental data. This relationship may be one of many types, but certain well-known characteristics of C are an aid in determining its character. First, C is approximately a constant. Furthermore, it is generally supposed that viscosity plays a much more important rôle than surface tension in determining C.

Assume that C is a function of **R** alone and independent of Weber's number, and that for large values of Reynolds' number (high heads and low viscosities) $1/\mathbf{R}$ approaches zero and C approaches a constant value C_1. When $C - C_1$

versus **R** is plotted logarithmically, the points fall along a straight line whose equation is of the form

$$C - C_1 = \frac{B_1}{\mathbf{R}^n} \qquad (11)$$

in which B_1 = a constant. Data from tests with water flowing over a 45° brass V-notch are shown in Fig. 8(*a*). Evaluation of constants shows $C_1 = 0.56$, $B_1 = 2.60$, and $n = 0.36$. Mr. Mawson (8) first expressed this relationship and gives an equation that can be reduced to the form of Eq. 11.

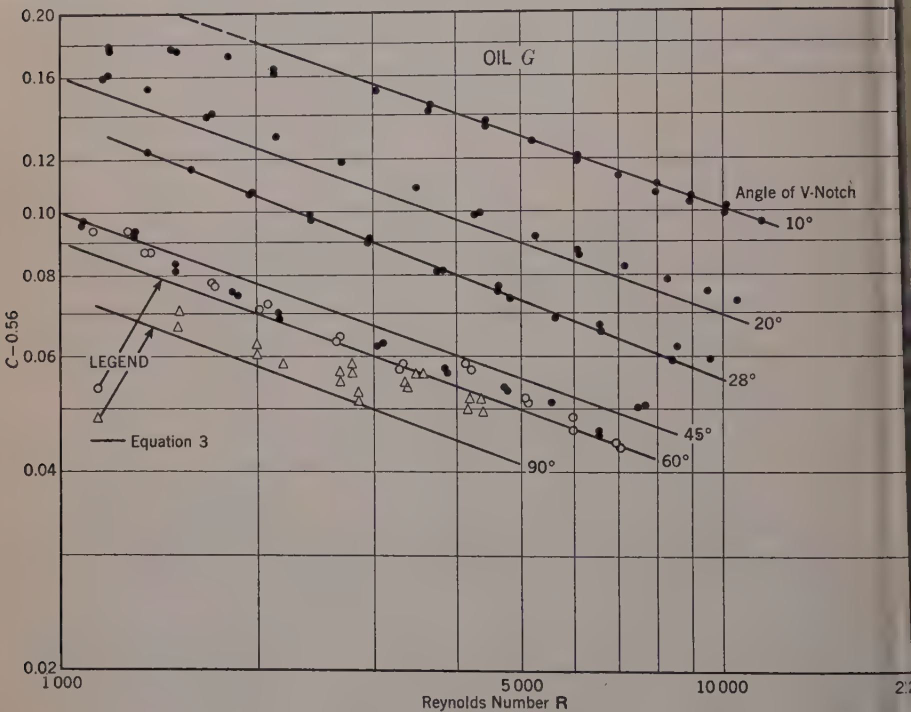


FIG. 11.—COEFFICIENT CU

Data from tests on a 28° V-notch for water and two oils are shown in Figs. 9 and 10. Again the straight-line relationship previously noted holds true for Reynolds' number greater than 850. Evidently the form of Eq. 11 is generally correct, but the assumption that C is independent of Weber's number is not correct.

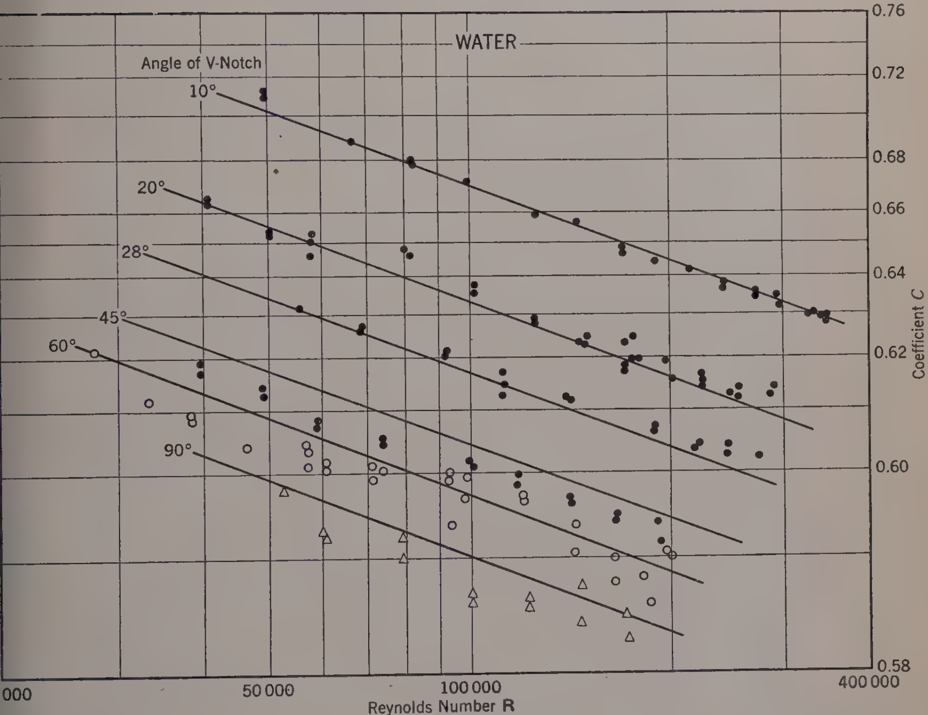
Plotting $C - C_1$ against **W** logarithmically again shows a straight-line relationship, the equation for the line being of the form:

$$C - C_2 = \frac{B_2}{\mathbf{W}^m} \qquad (12)$$

in which C approaches a constant value C_2, and B_2 = a constant. Experimentally it has been found that $C_2 = C_1$. Coefficients from tests shown in Fig. 8(*a*) are plotted against **W** in Fig. 8(*b*). Evaluation of the constants shows that $C_2 = 0.56$, $B_2 = 0.329$, and $m = 0.28$.

Both Eqs. 11 and 12 are those of a line on a plane, the plane being determined by the ordinate scale C and the abscissa scale **R** or **W**. It is evident, therefore, that the straight lines that are revealed on the C-**R** and C-**W** planes are traces of a line on an inclined plane surface whose three mutually perpendicular axes are C, **R**, and **W**. The equation for such a line on the inclined plane surface is

$$C - C_x = \frac{B_x}{\mathbf{R}^n\,\mathbf{W}^m} \quad \text{(13)}$$



ISCONSIN TESTS

The experimental data plot as straight lines on such inclined planes, one line for each liquid in each plane, and one plane for each angle of notch. Since it is difficult to show such three-dimension figures, the traces of these lines in the C-**R** plane are shown in Fig. 11. However, the position of any point depends on **W** as well as **R**. For a given angle of V-notch, there is only one value of coefficient for each set of **R** and **W** values.

Evaluation of the constant C_x in Eq. 13 has shown it to be equal to 0.56 for all angles of notch, at least through 90°. The general equation for the coefficient of discharge (see Eq. 3) is $C = 0.56 + \frac{B}{\mathbf{R}^n\,\mathbf{W}^m}$.

The values B, n, and m are functions of the angle of notch and may be

computed from the following equations or read from curves of these equations:

$$B = 0.475 + \frac{0.225}{\left(\tan\frac{\theta}{2}\right)^{0.80}} \quad \text{(14a)}$$

$$n = 0.165\left(\tan\frac{\theta}{2}\right)^{0.09} \quad \text{(14b)}$$

and

$$m = \frac{0.170}{\left(\tan\frac{\theta}{2}\right)^{0.035}} \quad \text{(14c)}$$

Reynolds' Number—Coefficient Curves.—Results of the Wisconsin tests with water and oil G are shown in Fig. 11 where **R** is plotted against $C - 0.56$. Plotted points indicate test values, and the lines that accompany each series of points are computed from Eq. 3. Fuel-oil tests at 70° F and 100° F were both above and below the critical range of Reynolds' number where Eq. 3 fails to hold true.

Limiting Conditions for Eq. 3.—The use of Eq. 3 is limited by the following boundary conditions: (*a*) Minimum Weber's number (minimum head), (*b*) minimum Reynolds' number, (*c*) minimum value of coefficient, and (*d*) minimum and maximum angle of notch.

(*a*) Minimum Weber's Number.—It is well known that below certain heads the flow over the notch ceases to jump free and clings to the downstream face of the weir plate. With water, heads less than 0.20 ft are to be avoided. However, it was possible to obtain good data for oils with heads as low as 0.12, indicating that clinging was probably a surface-tension effect. From examination of the data it was found that all tests with **W** less than 300 did not agree with Eq. 3. Therefore, this value is considered to be one limiting condition beyond which the equation should not be used. The necessary heads to give **W** = 300 are as follows:

Liquid	Head in ft	Liquid	Head in ft
Water	0.155	Oil G	0.106
Oil C	0.115	Oil A	0.105
Oil M	0.112		

The foregoing values are for 70° F, but change very little within the range of temperatures tested. The curves in Figs. 7 and 12 were stopped at these heads. It should be noted that if the jet clings to the face of the weir plate, Eq. 3 will not give correct results even if **W** is greater than the minimum value stated here. The minimum value of **W** appears to be unrelated to the angle of notch.

(*b*) Minimum Reynolds' Number.—Study of Fig. 13 (and similar curves for several values of θ) indicates that, below a certain minimum critical value of **R**, viscosity affects the flow, and consequently the coefficient, in a different manner than above that value of **R**. This change in coefficient is not to be confused with that which has been discussed under the heading "Minimum Weber's Number." For example, comparison of data in Fig. 13(*a*) for the 28° V-notch

shows that below **R** = 845, $C - 0.56$ definitely falls away from the line of Eq. 3. Although the break occurs at the same value of **R** for series at both 70° and 100° F, corresponding heads in the two series are 0.363 ft and 0.207 ft, respectively, and the corresponding values of **W** are 2,840 and 1,180, respectively. Similar results are evident in all the tests of oil A, although the value of **R** below which the viscosity effect changes is not the same for all angles. This critical value may be expressed by the equation

$$\mathbf{R} = \frac{300}{\left(\tan\frac{\theta}{2}\right)^{0.75}} \quad \ldots (15)$$

Clearly, the change in flow is due to a change in viscosity rather than changes in surface tension or head. It is also evident that the critical value of **R** is much lower than can be obtained with water under any practical test and therefore need not be considered in water measurement.

(c) Minimum Value of Coefficient.—For infinitely large values of **R** and **W** the coefficient C computed from Eq. 3 becomes equal to 0.56; yet no data obtained in these tests or any published have ever shown values that low.

The newest tests under conditions that give large values of **R** and **W** are the Cornell tests (7). These test values of $C = 0.56$ are plotted against **R** in Fig. 14. In all cases the coefficient was found to decrease to a minimum value and to remain approximately constant at that value for all greater values of **R**, independent of the angle of the V-notch. This constant value of C was 0.5886. The minimum C obtained by D. R. Yarnall (14) with his 90° notch was 0.5808.

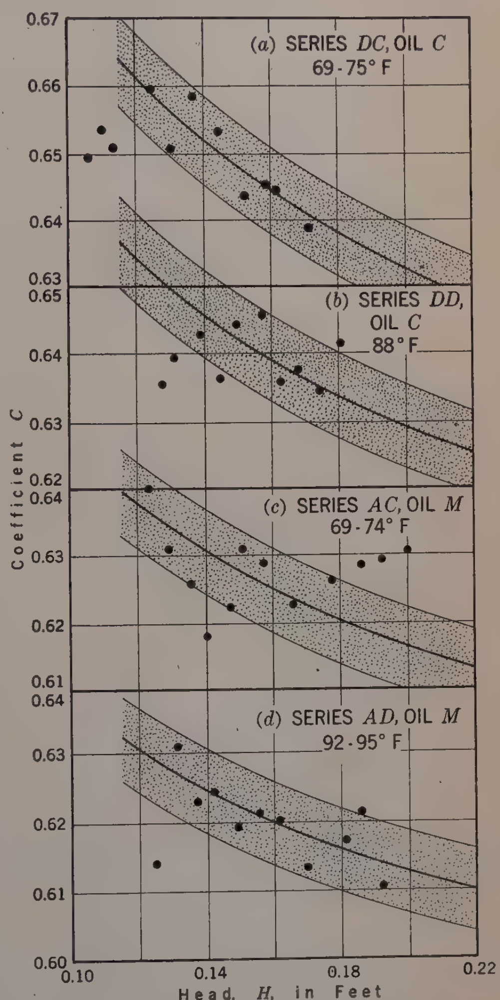


FIG. 12.—HEAD-COEFFICIENT CURVES; CALIFORNIA TESTS WITH A 90° V-NOTCH WEIR

Although it appears from the foregoing data that there is a limit to the applicability of Eq. 3, the reason for that limit is not obvious. It has been

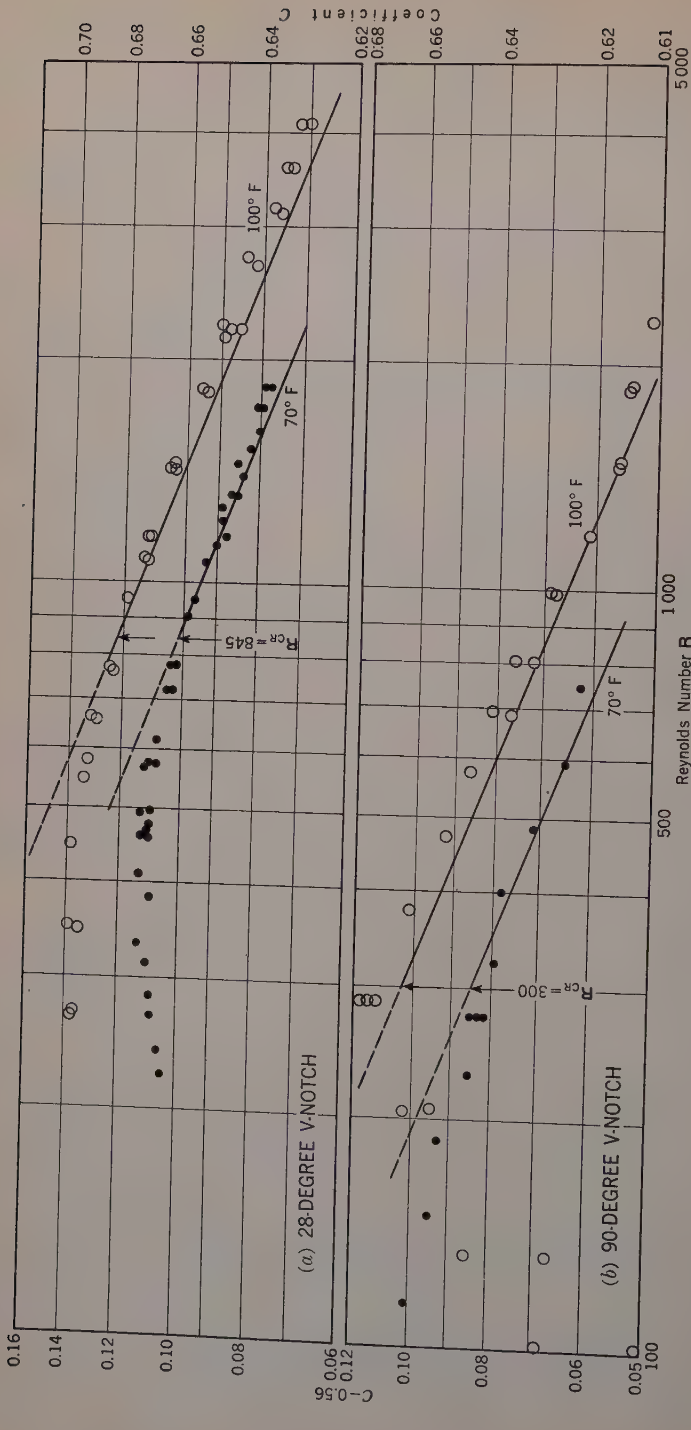


FIG. 13.—REYNOLDS' NUMBER—COEFFICIENT CURVES; WISCONSIN TESTS, OIL A

suggested by Messrs. Eaton (5) and Smith (11) that the ratios $\frac{H}{Z}$, $\frac{H}{b_w}$, and $\frac{L}{b_w}$ as given in Table 4 influence the velocity of approach in such a manner as to counteract the decrease in C as expressed by Eq. 3 (Z = height of the vertex of the notch and b_w = width of weir box). It appears from the data in Table 4

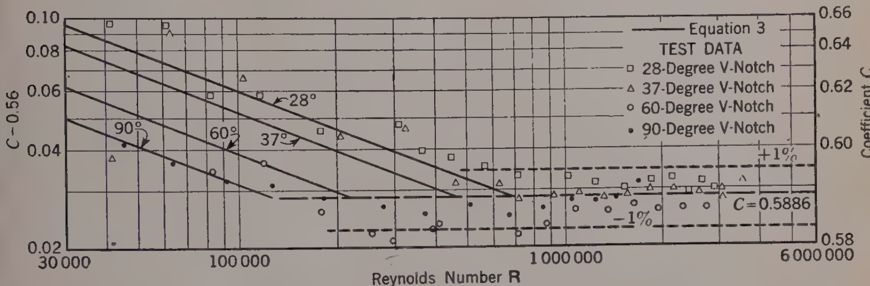


Fig. 14.—Reynolds' Number—Coefficient Curves; Cornell Test

that the experimental range of the three ratios is so great for constant C that there must be some factor other than the dimensions of the approach channel responsible for the constant coefficient. As further proof, it should be noted that in all cases the velocity head is a small proportion of the total head, and that the velocity itself, as measured by floats, at the head beyond which the coefficient is constant, is less than 0.08 ft per sec.

TABLE 4.—Range of Data (Cornell Tests) for Approximately Constant Values of Coefficient C

No.	Description	$\theta = 90°$		$\theta = 60°$		$\theta = 37°$		$\theta = 28°$	
		From:	To:	From:	To:	From:	To:	From:	To:
Average coefficient		0.5872		0.5856		0.5889		0.5915	
1	Head H (ft)	0.420	2.190	0.680	2.976	0.862	3.431	0.982	3.172
2	Discharge Q	0.287	17.95	0.552	22.12	0.579	18.39	0.604	11.31
3	L/b_w	0.1400	0.7300	0.1309	0.5727	0.0958	0.3812	0.0817	0.2640
4	Velocity head	0.0001(−)	0.0026	0.0001(−)	0.0050	0.0001(−)	0.0034	0.0001(−)	0.0009
5	H/Z	0.0525	0.2738	0.0850	0.3725	0.1078	0.4288	0.1228	0.3963
6	H/b_w	0.0700	0.3650	0.1133	0.4967	0.1437	0.5717	0.1637	0.5283
7	Reynolds' number	142,000	1,669,000	294,000	2,714,000	450,000	3,498,000	535,000	2,949,000

Mr. Eaton shows also a dimensionless ratio H/w, in which w is the thickness of the crest. If a minimum value of this ratio determines the head beyond which the weir coefficient is constant, this limiting value should be the same for both the Cornell and Yarnall tests with 90° weirs. It is not the same, being 40.3 and 246, respectively. Apparently H/w has no important bearing on the limit to Eq. 3, but w alone may have some bearing, as the minimum coefficient for $w = \frac{1}{8}$ in. (Cornell tests) was 0.5856; and for $w = \frac{1}{32}$ in. (Yarnall tests), 0.5808. Wisconsin tests, with $w = \frac{1}{32}$ in., approach the latter value.

In summary it should be stated that the coefficients decrease with increasing head in reasonably good agreement with Eq. 3 until a certain minimum coefficient is obtained. For all greater heads the coefficient is nearly constant for $w = \frac{1}{32}$ in. at approximately 0.581, which value is unrelated to θ, H, **R**, **W**, $\frac{H}{Z}$, $\frac{H}{b_w}$, $\frac{H}{w}$, or $\frac{L}{b_w}$.

Apparently, beyond certain minimum values of **R** and **W** as previously mentioned, the viscosity of the flowing liquid affects the internal fluid friction in the jet and the surface tension affects the shape of the jet in such a manner as to make Eq. 3 applicable. The coefficient decreases as the head increases until a change takes place, and there is no further decrease in C with all further increases in head, indicating that the change in flow with change in head is not dependent upon the viscosity or surface tension of the liquid.

(*d*) Minimum and Maximum Angle of Notch.—The Wisconsin tests reported in this paper were on angles varying from 10° to 90° inclusive, and show good agreement with Eq. 3 throughout the entire range.

For angles less than 28°, however, the Wisconsin tests do not agree with other published test data as follows—Eq. 3 is: (*a*) 3.2% below the Yarnall data for a 13° V-notch, (*b*) about the same amount below the Cone formula (4)

$$Q = (0.025 + 2.462\ s)\ H^{(2.5-\alpha)} \qquad (16)$$

which is not recommended for angles less than 28°, and (*c*) $3\frac{1}{3}$% below the Cornell data for a 19° V-notch. In Eq. 16, $s = \tan\frac{\theta}{2}$ = the slope of the side, and

$$\alpha = \frac{0.0195}{s^{0.75}} \qquad (17)$$

Eq. 16 has been reduced to the form

$$C = \frac{0.5758}{H^{\alpha}} + \frac{0.005845}{s\ H^{\alpha}} \qquad (18)$$

F. W. Greve (6) did not publish his data for a 10° notch because it was "not in harmony" with his other tests. When such disagreement exists among careful experimenters whose work agrees for the larger angles of notch, it would seem advisable to restrict the use of the V-notch to angles between 28° and 90° if good results are to be expected without an actual calibration of the weir in place.

One difficulty encountered in the use of narrow angles is the determination of the correct zero reading, because the geometrical point of the vertex is not likely to be at the actual metal point of the V. It may be noted in Fig. 15 that Eq. 3, the Cone formula, and Yarnall test data for the 13° V-notch are in closer agreement at high heads where an error in the determination of the zero reading is of less importance than at low heads.

Head-Discharge Equation for Water

When Eq. 3 is substituted for C in Eq. 2, the water discharge for a given angle of V-notch may be computed directly from the head because the water

temperature has very little effect on the coefficient. The combination of the two equations reduces to

$$Q = \left(2.395 + \frac{N}{H^e}\right) \tan\frac{\theta}{2} H^{2.5} \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

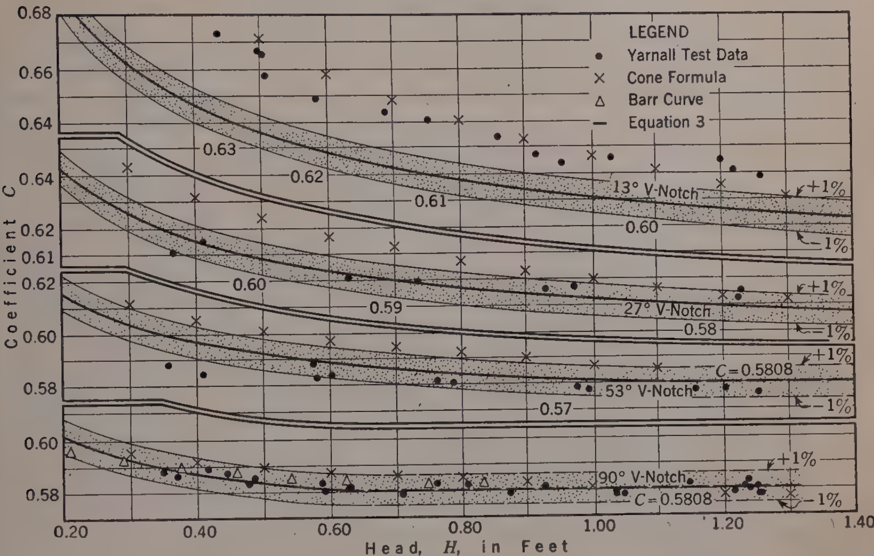


FIG. 15.—HEAD-COEFFICIENT CURVES; YARNALL, CONE AND BARR TESTS

Values of the constants N and e are given in Table 5 for water at 70° F. The discharge, Q, is in cubic feet per second, and H is in feet. The minimum value of the ratio $\frac{N}{H^e}$ is 0.090. For accuracy the head must be more than 0.155 ft, and the jet must not cling to the downstream face of the weir plate.

TABLE 5.—VALUES OF THE CONSTANTS N AND e (EQ. 19)

Constant	Angle θ: 90°	60°	45°	28° 04′	20°	10°
N	0.068	0.087	0.102	0.135[a]	0.167	0.267
e	0.588	0.582	0.579	0.575[a]	0.573	0.569

[a] Side slope 4 on 1 equals 28° 04′.

COMPARISON WITH PUBLISHED DATA

Water Tests.—Few liquids other than water have ever been measured over V-notches so that comparisons made with published data can reveal little about the effect of viscosity and surface tension on the coefficients; yet, if Eq. 3 is applicable for all liquids of reasonable viscosity, it must be applicable for water. Head-coefficient curves for water tests are shown in Fig. 15, in comparison with curves computed from Eq. 3, and demonstrate reasonable agreement for angles of 28° and larger.

The best known water tests on V-notches are those made by James Barr at Glasgow University in Scotland (2). He ran tests (2a) using a 90° V cut

in a 27-in. by 11-in. brass plate $\frac{3}{16}$ in. thick but "filed up" so that the breadth of the edge was reduced to about $\frac{1}{16}$ in. This group of his tests conforms most closely with the Wisconsin tests of Series 90D, and points from his test curve are plotted in Fig. 15 in comparison with Eq. 3, the Cone formula and Yarnall (14) test data. Above a head of 0.33 ft the Barr curve shows greater coefficients than those computed from Eq. 3, the maximum difference being about 0.75% as plotted; yet, if it is true that the Barr curve should decrease with head to a minimum value of $C = 0.5840$ and remain constant for all larger heads in agreement with boundary condition (*c*) (see under "Analysis of Data: Reynolds' Number—Coefficient Curves"), a not unlikely condition, as may be seen from Fig. 9 by reference to Mr. Barr's paper (2*a*), the maximum difference would be less than 0.5%.

The Yarnall data also agree well with Eq. 3, being within 0.5% of the line for all runs on the 90° notch and, except for a few runs, within 1% on both the 53° and 27° notches. Eq. 3 is slightly higher than the data for the 53° notch and slightly lower than the data for the 27° notch. The Yarnall data for the 13° notch are about $3\frac{1}{3}$% above Eq. 3 for the same notch, as previously discussed. It should be noted that, in computing **R** and **W** for these data, it was assumed that the temperature of the liquid was 65° F, which corresponds with a unit weight for water of 62.34 lb per cu ft as recorded.

The Cone formula (Eq. 16) was developed from tests on brass plates with $\frac{1}{16}$-in. or $\frac{1}{32}$-in. breadth of edge, the latter being the same breadth as used in the Wisconsin tests. This formula gives values of C greater than those computed by Eq. 3, although the agreement is within 1% for the 90° notch. However, except for the 13° notch, the Yarnall data are in closer agreement with Eq. 3 than with the Cone formula.

Within the range of applicability, Eq. 3 checks head-coefficient data for the Cornell tests fairly well, particularly for both the 28° and 90° notches. The data for the 60° notch were lower than computed, and for the 37° notch higher than computed. With the exception of a few runs, all the data were within 1% of Eq. 3 where it is applicable.

Data by Professor Greve on tests at Purdue University caused him to conclude (6*a*): "Apparently the coefficient is dependent upon head and independent of size [of notch]." The latter conclusion is contrary to the conclusions of this paper. His average coefficients for angles of V-notch between 20° and 120° are plotted in Fig. 16 in comparison with curves computed from Eq. 3 for V-notches of the Wisconsin tests (Series D). Professor Greve used soft steel plates as compared with the bronze used by Mr. Yarnall and brass as used by Messrs. Barr and Cone, and the writer.

Oil Tests.—The California tests on oil are the only ones that have come to the attention of the writer, on liquids other than water, and hence may be used as a check of Eq. 3 in so far as major viscosity and surface tension effects are concerned. Results of the California tests are plotted in Figs. 12 and 17. In each case the plotted points are from test data, and for comparison the line represents Eq. 3. It is of interest to note that, although they are rather variable (as would be expected at low heads), the coefficients are nearly all within 1% of Eq. 3; yet the variation in viscosity and surface tension in the

group of tests caused a 3.6% change in coefficient at a head of 0.18 ft, and the tests with the most viscous oil showed an increase in coefficient of 5.5% over that for water at the same head (0.18 ft). Since the surface tension of the oils tested varied as well as the viscosity, and since the ratio of surface tension to viscosity varied also, the agreement is considered satisfactory.

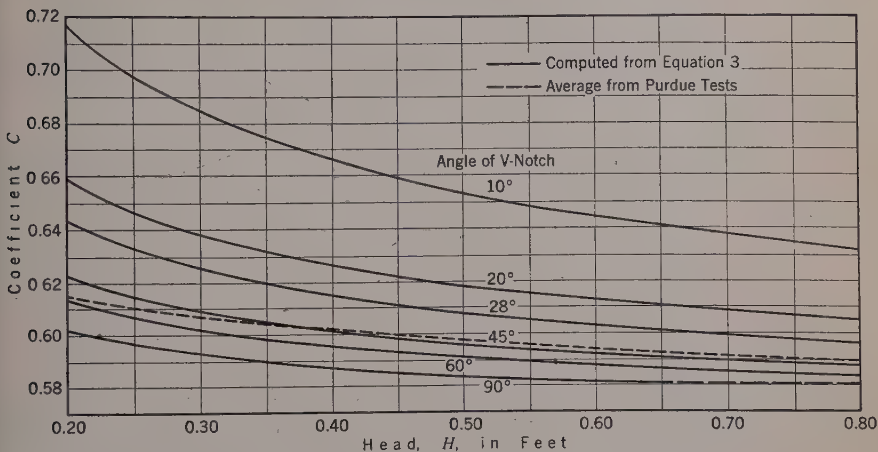


FIG. 16.—HEAD-COEFFICIENT CURVES FOR WATER

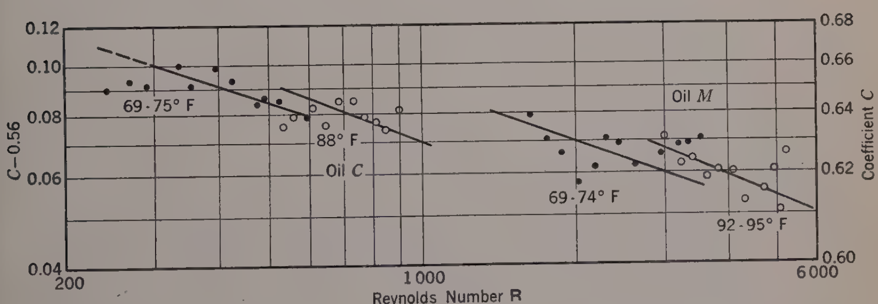


FIG. 17.—COEFFICIENT CURVES FOR CALIFORNIA TESTS ($\theta = 90°$)

Temperature Effect.—The effect of change of temperature on the flow of water over a V-notch is included in Eq. 3. Computations made for a head of 0.50 ft on a 90° weir give values of C as follows:

C	Temperature
0.5860	40° F
0.5840	70° F
0.5803	165° F

The coefficient decreases 0.34% between 40° and 70° F.

F. G. Switzer, M. Am. Soc. C. E. (12*a*), concludes that "the effect of water temperature is inappreciable compared with other factors present" after tests on 90° and 54° V-notches with water at 39°, 83°, 119°, and 165° F; yet he also states (12*a*) that "no point is more than 2 per cent from a mean and the great

majority of the points are within 1 per cent of the mean." Since the total change in C by Eq. 3 for this temperature range is but 1%, it is possible that the temperature effect may be hidden by other variables.

Conclusions

(1) The coefficient of discharge for a V-notch weir, C in Eq. 2, is a dimensionless function of Reynolds' and Weber's numbers. The coefficient may be computed from Eq. 3, in which Reynolds' number and Weber's number are defined by Eq. 9. The coefficients B, n, and m in these formulas have constant values for a given angle of notch. They may be computed from Eq. 14.

(2) Test data indicate that Eq. 3 is limited in its application as follows:

A. Lower Limits.—(*a*) The value of Weber's number must exceed approximately 300. For water the head should preferably be greater than 0.20 ft, and in any case the liquid must not cling to the downstream face of the weir plate. (*b*) The value of Reynolds' number must exceed a critical minimum value of approximately $\dfrac{300}{\left(\tan \dfrac{\theta}{2}\right)^{0.75}}$; Tests with water always exceed this minimum.

B. Upper Limit.—Coefficient C never will be less than a certain minimum, depending on the conditions of test. Minimum values for published test data are within 1% of 0.585. The lowest obtained was 0.5808 by Mr. Yarnall. Coefficient C remains constant for all values of $\mathbf{R}$ and $\mathbf{W}$ beyond that at which the minimum C occurs.

C. Limiting Angles.—Eq. 3 is applicable for angles at least 28° to 90°, inclusive, and has been found to hold true for 10° and 20° V-notches tested by the writer.

(3) A head-discharge equation for the flow of water over V-notches is given in Eq. 19.

(4) Comparison with data by other experimenters is satisfactory.

A.—Water test coefficients by Messrs. Barr, Yarnall, and Ho and Wu are nearly all within 1% of those computed from Eq. 3 for angles from 28° to 90°, inclusive.

B.—Oil-test coefficients obtained at the University of California are within 1% of those computed from Eq. 3, although in extreme cases the values are 5.5% greater than those for water at the same head.

(5) The change in viscosity and surface tension when the temperature of water is increased from 40° F to 70° F is sufficient to cause a decrease in the computed discharge coefficient of 0.34%. Increasing the temperature from 40° F to 165° F decreases the computed C about 1%.

Acknowledgments

The writer is indebted to F. M. Dawson, M. Am. Soc. C. E., formerly at the University of Wisconsin, who suggested the problem as a thesis subject[4] and

[4] "The Effect of Viscosity and Surface Tension Upon V-Notch Weir Coefficients," by Arno T. Lenz; thesis presented to the University of Wisconsin in 1940, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

gave many helpful suggestions. The work was completed under the direction of J. G. Woodburn, M. Am. Soc. C. E. S. E. Kotz built the apparatus for the runs of Series A and assisted in obtaining data for those runs. Two thesis students, J. R. Frost and A. J. Gollnick, assisted with the construction of the apparatus for the runs of Series D and G. Several students assisted with the collection of data under appropriations from the National Youth Administration.

The California data, applying only to 90° weirs, were obtained by students working under the direction of Professor O'Brien at the University of California, and were loaned to the writer by Professor O'Brien.

The description of the Cornell tests was based on the research of Messrs. Ho and Wu (7) under the direction of Professor Schoder.

APPENDIX

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics, prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:[5]

A = area of cross section;
B = a coefficient: B_1, B_2, $\cdots$ B_x = constant coefficients for any given weir angle;
b_w = width of channel;
C = V-notch weir coefficient: C_1 and C_2 = constant weir coefficients for large values of $\mathbf{R}$ and $\mathbf{W}$, respectively, equal to 0.56; and C_x = a constant weir coefficient for values of $\mathbf{R}$ and $\mathbf{W}$ equal to 0.56;
c = coefficient = $Q/h^{2.5}$;
e = a constant exponent for any given weir angle;
f = a function;
F = a force;
g = gravity constant;
H = hydraulic head, in feet;
L = a length;
m = a constant coefficient;
N = a constant coefficient for any given weir angle;
n = a constant coefficient;
Q = flow, in cubic feet per second;
$\mathbf{R}$ = Reynolds' number;
s = slope of the side of a weir;
t = time;
V = velocity = Q/A;
$\mathbf{W}$ = Weber's number;

[5] ASA–Z10.2–1942.

w = thickness of the crest of a weir;
Z = height of the vertex of a notch;
θ = notch angle;
μ = viscosity;
ν = kinetic viscosity;
π = π_1, π_2, etc. = independent products of the arguments V, H, ρ, etc.;
ρ = density of liquids; and
σ = surface tension.

Bibliography

(1) **"Tentative Method for Test of Kinematic Viscosity,"** A. S. T. M. Standard D445–39T, 1937; revised, 1938 and 1939.

(2) **"Experiments Upon the Flow of Water Over Triangular Notches,"** by James Barr, *Engineering*, Vol. 89, April 8, 1910, pp. 435–438, and April 15, 1910, pp. 470–473. (*a*) See Fig. 9.

(3) **"Dimensional Analysis,"** by P. V. Bridgman, Revised Ed., Yale Univ. Press, New Haven, Conn., 1931. (*a*) p. 40.

(4) **"Flow Through Weir Notches with Thin Edges and Full Contractions,"** by V. M. Cone, *Journal of Agricultural Research*, U. S. Dept. of Agriculture, Vol. 23, March, 1916, p. 1051.

(5) **"The V-Notch Weir for Hot Water,"** by Ed S. Smith, Jr., discussion by H. N. Eaton, *Transactions*, A. S. M. E., Vol. 57, 1935, pp. 249–250.

(6) **"Flow of Water Through Circular, Parabolic, and Triangular Vertical Notch-Weirs,"** by F. W. Greve, *Engineering Bulletin*, Purdue Univ., Lafayette, Ind., Vol. 40, March, 1932, pp. 1–84. (*a*) p. 33.

(7) **"The Flow of Water Over Sharp Crested Weir Notches, Rectangular, Trapezoidal, and Triangular,"** by Chitty Ho and Sze-Ling Wu; thesis presented to Cornell Univ. in November, 1931, in partial fulfilment of the requirements for the degree of Master of Civil Engineering.

(8) **"Applications of the Principles of Dimensional and Dynamical Similarity to the Flow of Liquids through Orifices, Notches and Weirs,"** by Hubert Mawson, *Proceedings*, The Institution of Mechanical Engrs., Vol. 1, 1927, p. 1033.

(9) **"International Critical Tables,"** National Research Council of the United States, 1st Ed., New York, N. Y., 1926.

(10) **"Precise Weir Measurements,"** by E. W. Schoder and the late K. B. Turner, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 999.

(11) **"The V-Notch Weir for Hot Water,"** by Ed S. Smith, Jr., *Transactions*, A. S. M. E., Vol. 56, 1934, pp. 787–789.

(12) **"Tests of the Effect of Temperature on Weir Coefficients,"** by F. G. Switzer, *Engineering News*, Vol. 73, April 1, 1915, pp. 636–637. (*a*) p. 636.

(13) **"On Experiments on the Measurement of Water by Triangular Notches in Weir Boards,"** by J. Thomson, Annual Rept., British Assn. for the Advancement of Science, 1858, pp. 181–185; 1861, pp. 151–158.

(14) **"Accuracy of the V-Notch Weir Method of Measurement,"** by D. R. Yarnall, *Transactions*, A. S. M. E., Vol. 48, 1926, pp. 939–964.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

HYDRODYNAMICS OF MODEL STORM SEWER INLETS APPLIED TO DESIGN

By G. S. Tapley,[1] Assoc. M. Am. Soc. C. E.

Synopsis

A technique is developed in this paper by which it is possible to apply experimental results from $\frac{3}{32}$-scale models of side-opening storm sewer inlets to the design of full-size prototypes. The method is applied to design in a single case, the development and presentation of design data for various other shapes, slopes, and dimensions being reserved for the future.

The phenomenon of flow into an inlet is characteristically gravitational in type, but the small depths of flow in the model suggest that the effect of viscosity is also important. Lateral deflection of the flow is an additional characteristic.

An equation was thus sought and obtained describing the portion of the flow entering the inlet in terms of Froude's and Reynolds' numbers and the moment of pressure and moment of momentum. The development of the form of the equation was facilitated by an analytical determination of boundary conditions corresponding to infinite and zero approach velocity.

Total differentiation yielded an expression in which Reynolds' number has an arbitrary value with respect to viscosity. A Reynolds' number of prototype magnitude, but with model velocity and linear dimension, was thus selected, the value of the coefficient of viscosity being computed. Integration by means of planimeter and the use of flow-distribution relationships yielded a result which represents flow in the model similar to flow in a prototype with regard to both Froude's and Reynolds' laws.

In developing the basic data important facts regarding flow distribution in the channel upstream from the inlet were discovered.

Notation.—The letter symbols in this paper are defined where they first appear and are assembled for reference in Appendix B.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by July, 1942.

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I.—General

The action of prototypes can often be predicted from the observed operation of models together with the application of a single law of similarity, such as that of Froude or Reynolds. In other instances, such as in open-channel flow and ship-model work, no single law can be considered as being exclusively applicable.

The present case deals with the development of a technique for applying to the design of prototypes the experimental results from $\frac{3}{32}$-scale models of side-opening storm-sewer inlets of a type shown in Fig. 1. Flow approaching the inlets is turbulent and gravity is the principal force; Froude's law evidently is important. On the other hand, depths of flow in the model varied from about

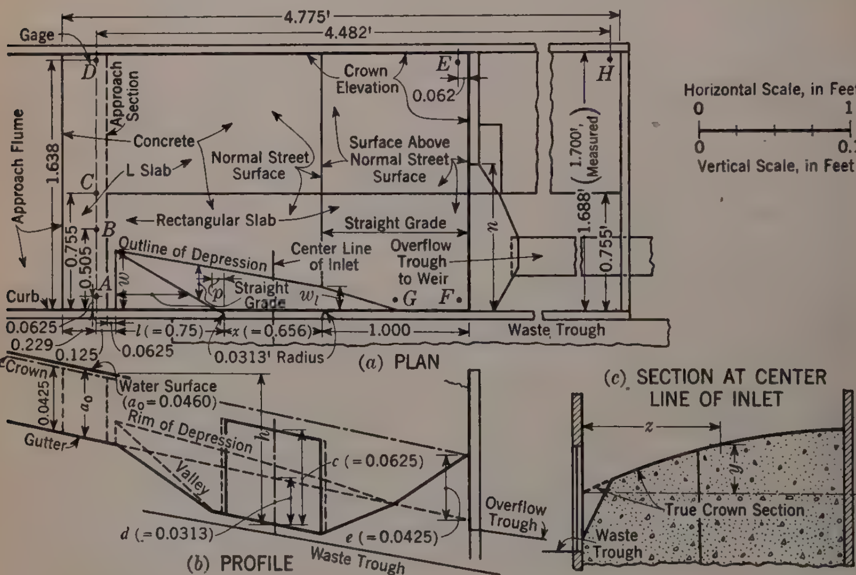

Fig. 1.—Storm-Sewer Inlet; 3/32-Scale

$\frac{1}{2}$ in. at the curb to film thickness at the outer edge of the stream; serious doubts would exist as to the validity of results in which the effect of Reynolds' number had been ignored. Flow approaching the inlets (except on streets with slopes flatter than about 0.005 ft per ft) is further characterized as being of the shooting type (velocity is greater than wave velocity). The flow, moreover, is wide in relation to depth and undergoes a lateral deflection in entering the inlet; a change of angular momentum is involved.

The development of technique described in this paper makes use only of the data necessary. Development of the remaining experimental data covering a range of shapes of depression, slopes of street, lengths of inlet, etc., remains for the future. The data utilized include only cases involving "free flow" at the lower end of the inlet. In this type of flow that portion of the water, if any,

(*a*) No Backwater Effects

(*b*) Considerable Backwater Effect

Fig. 2.—Views of Full-Size Inlets Showing Relative Backwater Effects

which passes the lower end of the inlet has a velocity greater than wave velocity. Free flow is differentiated from "backwater effect," a condition that occurs when a backwater wave extends upstream of the lower end of the inlet. Fig. 2 shows two views of actual full-size inlets. The inlet in Fig. 2(*a*) has no local depression. There is a small gutter rise below the inlet, and no backwater effect. The flow, Q_t, is 0.9 cu ft per sec. In Fig. 2(*b*), on the other hand, there is a local depression and a 6-in. or 8-in. gutter rise below the inlet. Considerable backwater effect is evident. Fig. 3 shows four views of the model with various stages of backwater effect. For example, Fig. 3(*a*) is a view facing diagonally downstream toward the inlet. The approach flume is in the foreground, and the overflow notch is directly below the hammer. Fig. 3(*b*) represents flow conditions for a heavy backwater effect; and the limiting conditions for backwater effect, with the wave at the lower end of the inlet, are shown in Fig. 3(*c*). When the backwater wave occurs downstream from the inlet, its effect misses the inlet, as shown in Fig. 3(*d*).

The local depression or approach apron adjacent to the inlet is of a type adopted on a basis of experiments made in 1926 and 1927.[2]

Under the wide variety of conditions encountered in practice the accumulation of a number of isolated full-size measurements of capacity is practically worthless, and the advantage of a small model must be evident. The recentness of the development of model experimental technique, no doubt, accounts, in part, for the fact that no satisfactory solution of the problem (so far as is known) has been obtained heretofore. With an estimated \$50,000,000[3] to be spent for storm drains in the vicinity of the City of Los Angeles, Calif., alone over a period of years, the economic importance of the problem is apparent. Moreover, a reexamination of the designs of existing installations probably will disclose instances in which the benefit of a part of investments already made in conduit systems is not being realized because of insufficient inlet capacity.

II.—Apparatus and Technique

The principal apparatus consisted of a tilting framework (permitting variations of slope from zero to 10%) supporting a stilling tank, an approach flume, and the model depression and inlet. The plan, profile, and cross section of the latter are shown in Fig. 1. The approach flume was 10 ft long. (This length was sufficient to insure "developed" flow at the depression as evidenced by a series of tests.) It represented, to scale, one half of a parabolic-crowned pavement, 36 ft wide, having a crown height of 0.45 ft. A false curb prevented water from spilling out of the flume at the crown side. The equation of the model cross section is

$$(z - 1.688)^2 = -67.0\,(y - 0.0425) \qquad (1)$$

in which y and z are as shown in Fig. 1. The flume was paved with a patented wallboard of glazed finish, which was nailed to templates cut to the proper section. The whole was made waterproof with spar varnish. Adjustments,

[2] See "Increasing Stormwater Inlet Capacity by Improved Gutter Approaches," by L. W. Armstrong and G. S. Tapley, *Engineering News-Record*, July 9, 1931, pp. 54–56.

[3] *Los Angeles Times*, June 10, 1939, p. 6.

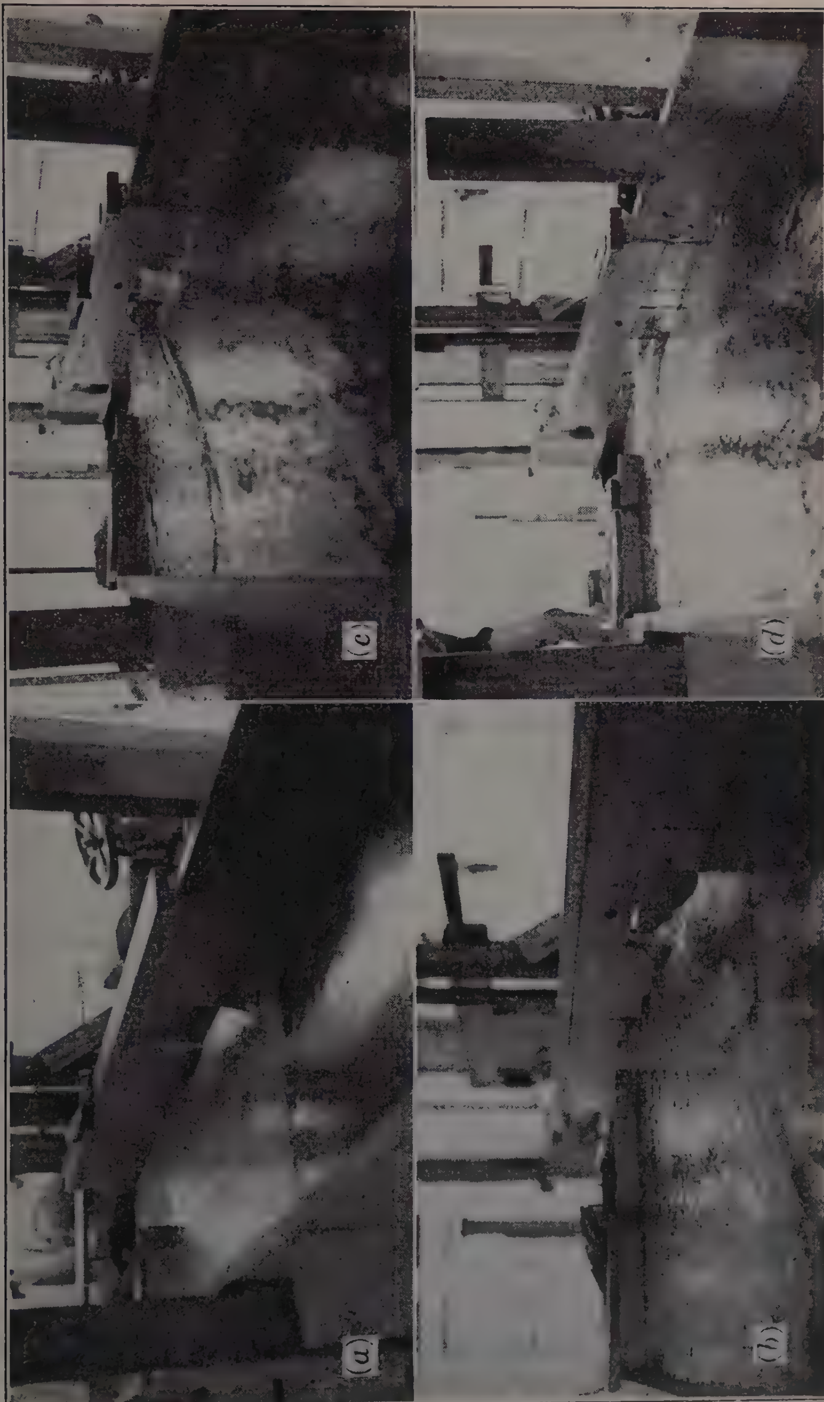

Fig. 3.—Views of Model Inlet Demonstrating Backwater Effects

for warp and sag in the flume, were made by means of the adjusting screws and wedges supporting the flume on the framework. Local adjustments in the flume pavement were made by sandpapering high areas and filling in low ones with lacquer putty or lacquer.

The pavement of the lower end of the apparatus which contained the depression and inlet was formed from two concrete slabs. The outer, L-shaped, "permanent" slab was supported on three adjusting screws attached to the tilting framework. These screws adjusted the slab to a slope different from that of the flume, when necessary, thus varying the approach velocity in the experiments. The L-shaped slab contained the orifices for gages A, B, C, D, and E. The slab itself was formed to normal section throughout, the part having a surface above normal (Fig. 1) being built up with plaster of Paris. This was chipped off and rebuilt when models of different length were installed. Gage E, Fig. 1, in such cases was also moved to keep the standard distance from the endboard. All joints were kept smooth and watertight with a mixture of beeswax and resin.

The other slab forming the pavement of the lower end of the apparatus was a rectangular one containing the depression. This slab was supported on three adjusting screws attached by brackets to the L-slab. The slab was so formed that the gutter line rises to crown elevation at a point 12 in. downstream from the lower end of the inlet. (The crown elevation is 0.0113 ft higher than the normal elevation, which corresponds to an actual street with its curb face 8 in. normally and 4 in. at the center of the curb return.) For the present case—free flow—the contour of the pavement below the inlet has no effect on flow entering the inlet. Indeed, the contour in that region has little influence on flow entering the inlet even in case of backwater effect, the principal factor then being the elevation of the backwater surface.

Depth gages A, B, C, D, E, F, and G, Fig. 1, consisted of copper tubes, $\frac{3}{16}$ in. inside diameter, whose orifice ends were carefully set flush with the surface of the concrete, and which were connected to stilling wells where the water surface elevations were read by means of a point gage. The zero-flow readings of the gages were obtained by wiping water from the orifices until the menisci were perfectly flat, thus avoiding the effect of surface tension. Usually an average of at least three readings of the zero points during a day's run was obtained.

Total flow, Q_t, approaching the inlet was measured by means of an orifice meter calibrated by weighing. The excess flow, Q_e, or flow passing the inlet was measured by means of a 15° V-notch weir (also calibrated by weighing) and the inlet flow was found by subtraction.

Data on horizontal Q-distribution were obtained in a separate series of experiments by intercepting the flow in the normal channel between thin vertical vanes set 2 in. apart (except near the crown where the spacing was wider) and volumetrically measuring the flow between each pair of vanes over a given time interval. The depths necessary for computing velocities from the discharges were measured by gages A, B, C, and D, Fig. 1.

The procedure for leveling the apparatus was first to set the flume by means of grade marks near the upper and lower ends of the flume, then carefully to level the L-slab into approximate agreement with the flume. The next step

was to adjust the lower end of the flume so that the joint was flush, ignoring the small departure from the lower flume marks. Adjustments in leveling the concrete were made until the difference in elevation between gages A and D, and between D and H, was correct to within 0.001 ft. This quantity was used as a criterion for acceptable accuracy throughout the model and lower end of the approach flume.

Readings of the gages were taken at time intervals of sufficient length to insure the existence of steady conditions, the weir being the controlling factor. Five minutes between weir readings was found to be a satisfactory interval. The maximum total flow in each set of readings was about 0.1 cu ft per sec, and the minimum about 0.02 cu ft per sec. The usual procedure was to turn on a flow of about 0.12 cu ft per sec for about 5 min, decrease the flow to about 0.10 cu ft per sec, wait for 2.5 min, locate the valley wave (a wave near the valley), read the orifice meter, read gages A, B, C, D, and E, and last, read the weir gage. Flow was then decreased in 6 or 8 stages at 5-min intervals to the minimum discharge. By this procedure the uneven effect of capillarity and surface tension on previously unwetted surfaces was avoided.

III.—Theory

For a fixed depth, a_0, and a known flow distribution in the channel upstream from the inlet, the ratio

$$K = \frac{\bar{z}\,P}{U} \qquad (2)$$

describes the flow within a distance z_i from the curb and represents a relationship between pressure and velocity patterns. In Eq. 2, $\bar{z}\,P$ is the moment of pressure, and U is the moment of momentum per second, or angular momentum per second, at the approach section, taking moments about the curb, corresponding to a filament of flow within a distance z_i from the curb.

The nature of K may be purely "distributional," or without regard to inlet flow; or it may be "gravitational," in which case the outer boundary of the filament of flow included coincides with the outer boundary of the inlet flow. In the latter case it must be evident that the value of K will depend upon the forces that act on the fluid as it passes from the approach section to the inlet lip. These forces may be described by means of a function of Froude's and Reynolds' numbers, and shape factors:

$$K = \phi\,(\bar{v}, h, \rho, g, \mu, a_0, d, t, c_s) \qquad (3a)$$

in which $\bar{v}\left(= \frac{G}{\rho\,Q}\right)$ is the average velocity of the water particles within a distance z_i from the curb; G = momentum per second; ρ = mass per cubic foot; and h is the fall of the water in passing from the approach section to the inlet lip. (The approach section was located 0.0625 ft upstream from the local depression, and thus a small part of the total fall represents fall in the approach channel. Depths were measured at a section 0.125 ft upstream from the depression. For cases in which the approach flume had a slope different from that of the model or the depth in the approach flume was greater than critical depth a small error in h is present.) Standard dimensional-analysis procedure

now yields

$$K = \psi(\mathbf{F}, \mathbf{R}, A_0, D, T, C_s) \ldots\ldots (3b)$$

in which Froude's number is

$$\mathbf{F} = \frac{\bar{v}^2}{g\,h} \ldots\ldots (4a)$$

Reynolds' number:

$$\mathbf{R} = \frac{\bar{v}\,h}{\nu} \ldots\ldots (4b)$$

the approach-depth number:

$$A_0 = \frac{a_0}{h} \ldots\ldots (4c)$$

the depth-of-depression number:

$$D = \frac{d}{h} \ldots\ldots (4d)$$

the roughness number:

$$T = \frac{t}{h} \ldots\ldots (4e)$$

and

$$C_s = \frac{c_s}{h} \ldots\ldots (4f)$$

which is the number representing that part of h due to the normal slope of the model.

Variations in the ratio A_0 cause changes in the geometrical shape of the

TABLE 1.—Experimental Data Required for an Analysis of Flow

(The Approach Depth a_0 = 0.0460; and Area = 0.0298)

Symbol	Remarks	Longitudinal Slope, S_f, of Approach Flume:					
		0.01	0.02	0.04	0.06	0.02	0.02
d	....	0.0313	0.0313	0.0313	0.0313	0.0	0.0625
h	....	0.1001	0.1001	0.1001	0.1001	0.0688	0.1313
Q_t	Fig. 4	0.0436	0.0613	0.0864	0.1063	0.0613	0.0613
Q_e	Fig. 5	0.0197	0.0377	0.0617	0.0835	0.0487	0.0236
Q	$Q_t - Q_e$	0.0239	0.0236	0.0247	0.0228	0.0126	0.0377
Q/Q_t	....	0.549	0.385	0.286	0.214	0.206	0.615
z_i/z_t	Fig. 6	0.217	0.141	0.102	0.076	0.072	0.252
z_i	$(z_i/z_t) \times z_t$	0.367	0.239	0.172	0.129	0.122	0.4265
Q^2_t	....	0.00190	0.00375	0.00747	0.01130	0.00375	0.00375
G_1	Fig. 7	42.9	30.1	22.2	16.8	15.7	47.7
G	$G_1 \times Q^2_t$	0.0816	0.1129	0.1657	0.191	0.0589	0.1790
$\bar{v}$	$G/\rho\,Q$	1.761	2.468	3.461	4.31	2.412	2.447
v'	Q_t/area	1.46	2.05	2.89	3.55	2.05	2.05
$\mathbf{F}$	$\bar{v}^2/g\,h$	0.963	1.892	3.720	5.81	2.632	1.421
$1/\mathbf{F}$	....	1.038	0.5285	0.2688	0.172	0.3797	0.703
$\bar{z}\,P$	Fig. 8	0.00254	0.00132	0.00076	0.00043	0.00039	0.00312
U_1	Fig. 7	7.47	3.51	1.93	1.11	1.00	9.27
U	$U_1 \times Q^2_t$	0.01419	0.01316	0.0144	0.01254	0.00375	0.03472
K	$\bar{z}\,P/U$	0.1790	0.1003	0.0527	0.0343	0.1040	0.0899
$\nu \times 10^6$	....	14.9	13.7	14.37	14.37	12.7	13.3
$\mathbf{R}$	$\bar{v}\,h/\nu$	(11,810)	(18,030)	(24,040)	(29,970)	(13,070)	(24,150)
$K\,\mathbf{F}$	....	0.172	0.189	0.196	0.199	0.274	0.128
a_i	....	0.0296	0.0349	0.0379	0.0397	0.0401	0.0273
$a_i\,g/2\,\bar{v}^2$	....	0.153	0.0919	0.0508	0.0343	0.111	0.0731
$(1/\mathbf{F})^{0.5}$	....	1.017	0.727	0.518	0.415	0.616	0.838
$(1/\mathbf{F})^{1.5}$	....	1.062	0.383	0.139	0.0712	0.233	0.589
$(1/\mathbf{F})^2$	....	1.08	0.279	0.0723	0.0297	0.144	0.494
$a_i/2\,h$	....	0.148	0.1745	0.1895	0.1985	0.292	0.1041
$\bar{v}/Q_t$	....	40.4	40.3	40.1	40.5	39.3	39.9
$Q_t - Q$	....	0.0197	0.0377	0.0617	0.0835	0.0497	0.0236

approach-flow cross section. Analysis can best be made, therefore, by covering the desired range of values of the ratio in a series of steps, each step of which represents a constant value of A_0

Moreover, c_s is constant so long as S, x, and l are constant. When, also, d is constant, h is constant, for,

$$h = a_0 + d + c_s \qquad (5)$$

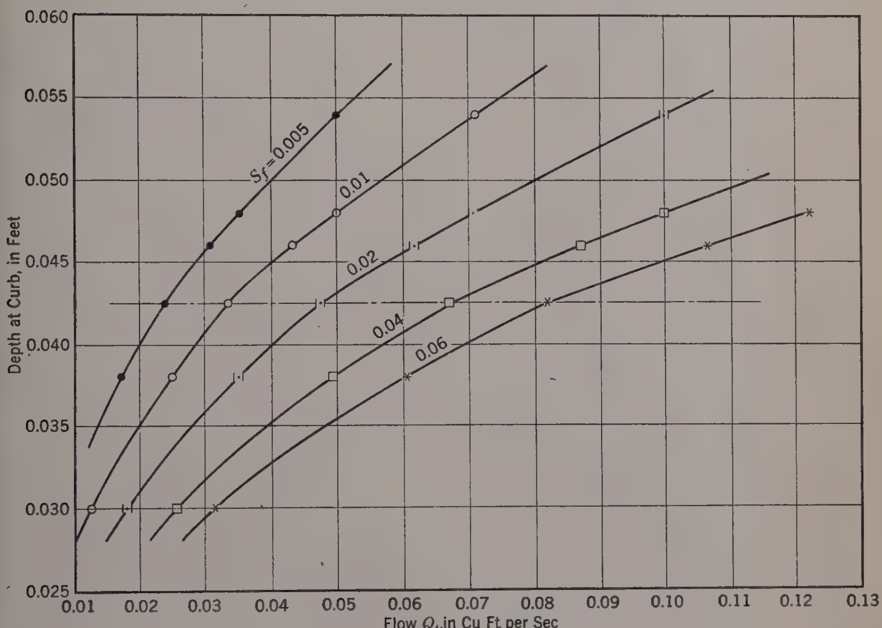

FIG. 4.—DEPTH AND DISCHARGE DATA

The magnitude, t, of the roughness protuberances was not varied in the model. (The results in this case apply to a prototype whose roughness particles are geometrically similar to those of the model. When, in the model, h is varied by varying d or a_0 the relative roughness varies although t itself is constant.) Under these conditions Eq. 3*b* reduces to

$$K = \theta(\mathbf{F}, \mathbf{R}) \qquad (6a)$$

or, in product form,

$$K = C_{AD}\, \alpha(\mathbf{F})\, \beta(\mathbf{R}) \qquad (6b)$$

This cannot be reduced further to simpler form, since (in the absence of varying ν or g) **F** and **R** vary simultaneously. The next step after preparing data in suitable form is to insert values in the equations and determine the forms of the functions, $\alpha(\mathbf{F})$ and $\beta(\mathbf{R})$.

IV.—PREPARING THE DATA

The experimental data required for an analysis of flow at the approach depth $a_0 = 0.0460$ ft are given in Table 1. Comparison can be made in

the table between the average velocity $\bar{v}$ and the apparent average velocity $v'_t = \frac{\text{total discharge}}{\text{total area}}$.

Values of Q_t in the table were taken from Fig. 4 which represents the averaged result of a large number of depth, Q_t, observations. The averaging of

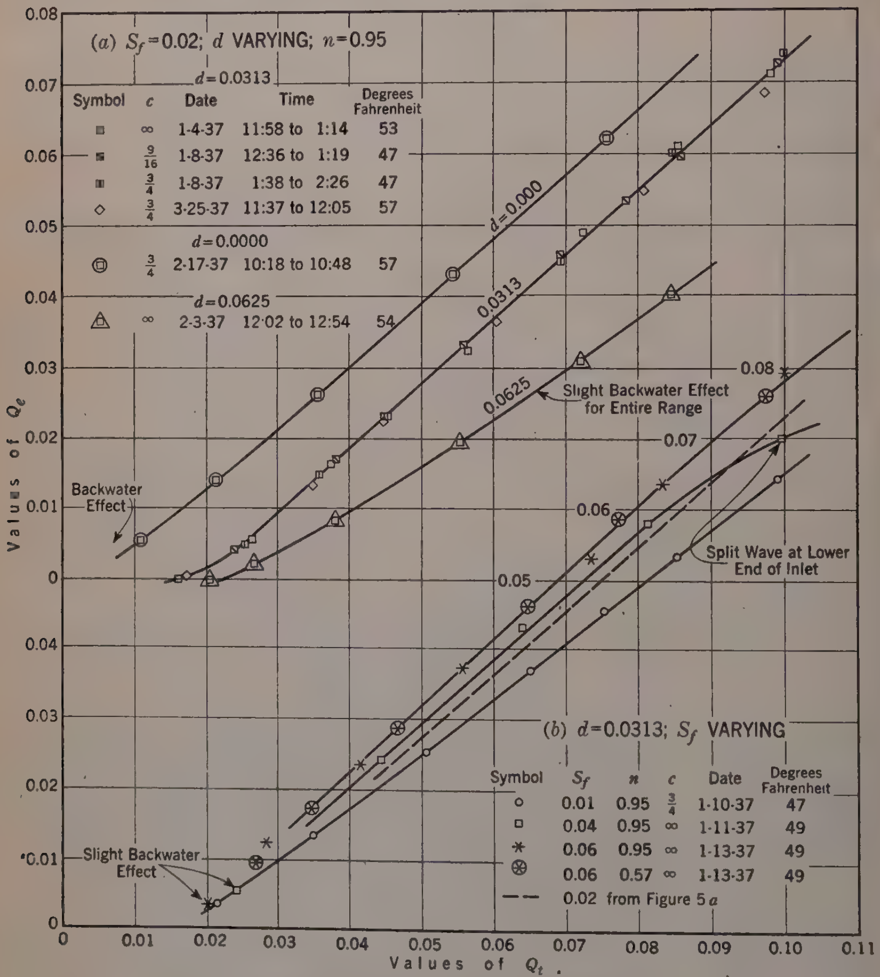

FIG. 5.—EXPERIMENTAL FLOW RELATIONSHIPS

depth-Q_t data was made necessary by the considerable dispersion of values. In the averaging process a cross slope of the water surface, which seemed to be systematic and to increase with S_f, the slope of the approach flume, was eliminated, and the resulting surface was made transversely level. The consistency of relationships described subsequently indicates acceptable accuracy of the resulting average. Slight discrepancies exist between tabular values of

Q_t and those of Fig. 4 because of recomputing and redrafting the figure. Slight backwater effect existed in the cases: $d = 0.0313$, $S_f = 0.01$; and $d = 0.0625$, $S_f = 0.02$. The process of obtaining the average depths in Fig. 4 is described in a comprehensive manuscript entitled "Depth and Distribution Data for Flow in Model Street," by the writer. This material, with supporting experimental data, has been placed on file in the Engineering Societies Library.[3a] The latter manuscript also contains the derivation of the distribution of Q/Q_t (Fig. 6) and hydrodynamic quantities shown in Figs. 7, 8, 16, and 17.

Values of Q_e in Table 1 may be taken from Fig. 5(a), representing a constant value of S_f and three values of d, and from Fig. 5(b), representing a constant value of d and various values of S_f selected to vary the approach velocity. The slope, S, of the model itself was held constant for the present set of data. The small dispersion of data representing the several separate settings of the apparatus indicated by the different symbols along the curve for $d = 0.0313$ in Fig. 5(a) indicates the good consistency which was characteristic of such data. The apparatus was set at a large number of different slopes and adjustments in the period between the two extreme dates given in Fig. 5(a).

Values of Q, the inlet flow, were obtained by subtracting values of Q_e from those of Q_t. Values of $\frac{Q_t}{Q}$ were computed. The values of $\frac{Q}{Q_t}$ in Table 1 represent gravitational quantities as distinguished from distributional ones.

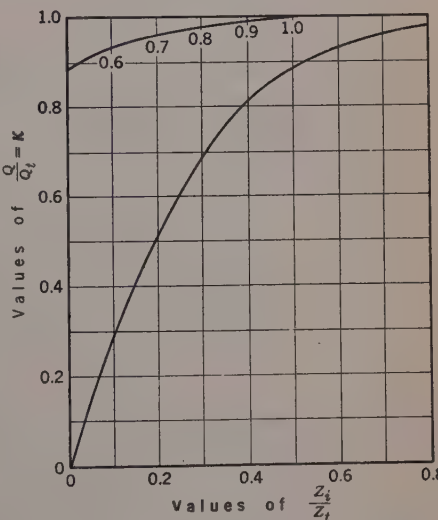

FIG. 6.—DISTRIBUTION VALUES OF Q/Q_t FOR $a_0 = 0.0460$

Fig. 6 shows distributional values of $\frac{Q}{Q_t}$ for $a_0 = 0.0460$. The graph has been made dimensionless by using values of the ratio $\frac{z_i}{z_t}$ for abscissas. In working up data for the figure it was found that the ratio $\frac{Q}{Q_t}$ for a given value of $\frac{z_i}{z_t}$ is constant for all values of Q_t to within a very small limit of error. $\left(\text{It is also true that } \frac{v}{Q_t} \text{ is constant for constant values of } a.\right)$ Hence, let

$$\frac{Q}{Q_t} = \kappa \qquad (7)$$

a constant for a given value of z_i.

[3a] 29 West 39th Street, New York, N. Y.

The consistency found in the ratio $\frac{Q}{Q_t}$ for different values of Q_t and given values of a_0 confirms, to some extent, the accuracy of the depth-Q_t results obtained by the averaging process mentioned. The fact that the value of the ratio is known for all values of Q_t, and hence for $Q_t = 1.000$, makes it possible to use a single set of distributional curves in calculations.

Designating by the subscript 1 quantities corresponding to $Q_t = 1.000$, it will be shown that the required quantities G and U can be expressed in terms of G_1 and U_1, respectively, and $Q^2{}_t$. It is convenient to introduce at this point distributional expressions for the various quantities:

$$G = \rho \int_0^{z_i} v^2\, a\, dz \qquad (8)$$

In Eq. 8, v represents the velocity at any value of z within the limits of the integral, a is the depth at that point, and z_i is the distance from the curb to the outer boundary of the filament of flow considered. (Actually v represents the apparent average velocity in a vertical line and not the velocity at a point. The integral—Eq. 8—therefore does not strictly represent the momentum of all the particles. It was deemed to be permissible to ignore vertical variation of velocity.) The value of z_i corresponds to the outer boundary of the inlet flow when the quantity represented by the integral is a gravitational one. Otherwise z_i refers to any desired point in the channel. Continuing with distributional expressions:

$$Q = \int_0^{z_i} v\, a\, dz \qquad (9a)$$

$$\bar{v} = \frac{G}{\rho\, Q} = \frac{\int_0^{z_i} v^2\, a\, dz}{\int_0^{z_i} v\, a\, dz} \qquad (9b)$$

$$\mathbf{F} = \frac{\bar{v}^2}{g\, h} = \frac{1}{g\, h}\left(\frac{\int_0^{z_i} v^2\, a\, dz}{\int_0^{z_i} v\, a\, dz}\right)^2 \qquad (9c)$$

$$\mathbf{R} = \frac{\bar{v}\, h}{\nu} = \frac{h}{\nu}\,\frac{\int_0^{z_i} v^2\, a\, dz}{\int_0^{z_i} v\, a\, dz} \qquad (9d)$$

$$U = \rho \int_0^{z_i} z\, v^2\, a\, dz \qquad (9e)$$

$$\bar{z}\, P = \frac{\gamma}{2}\int_0^{z_i} z\, a^2\, dz \qquad (9f)$$

By Eq. 7, $Q = \kappa Q_t$; so that

$$\frac{dQ}{dz} = Q_t \frac{d\kappa}{dz} \quad \text{(10)}$$

A differential of discharge can be written

$$dQ = v\, a\, dz \quad \text{(11)}$$

It follows, then, that

$$v = \frac{1}{a}\frac{dQ}{dz} \quad \text{(12a)}$$

Hence, by Eq. 10,

$$v = \frac{1}{a} \times Q_t \frac{d\kappa}{dz} \quad \text{(12b)}$$

in which the quantity

$$\frac{d\kappa}{dz} = \frac{d\left(\dfrac{Q}{Q_t}\right)}{dz} \quad \text{(13)}$$

represents the slope of a graph like Fig. 6, but having z_i instead of $\frac{z_i}{z_t}$ for abscissas. The value of v in Eq. 12b can thus be determined by a process that is theoretically exact. The accuracy of the result depends only on the accuracy of the data and the accuracy with which the slopes of the graph can be scaled. Therefore, the method does not involve the systematic error which results when, for example, it is assumed that the velocity at the midpoint between a pair of the vanes used in the distribution experiments referred to equals the increment of discharge divided by the increment of area between the vanes.

Putting the value of v from Eq. 12b in Eq. 8, keeping Q_t outside the integral sign (since Q_t is not a function of z) and canceling a, the result is

$$G = \rho\, Q^2{}_t \int_0^{z_i} \frac{1}{a}\left(\frac{d\kappa}{dz}\right)^2 dz \quad \text{(14)}$$

The quantity under the integral sign is purely a function of z so that, when $Q_t = 1.000$, Eq. 14 becomes

$$G_1 = \rho \int_0^{z_i} \frac{1}{a}\left(\frac{d\kappa}{dz}\right)^2 dz \quad \text{(15a)}$$

Hence,

$$G = G_1\, Q^2{}_t \quad \text{(15b)}$$

From Eqs. 12b and 9e,

$$U \doteq \rho\, Q^2{}_t \int_0^{z_i} \frac{z}{a}\left(\frac{d\kappa}{dz}\right)^2 dz \quad \text{(16a)}$$

so that

$$U = U_1\, Q^2{}_t \quad \text{(16b)}$$

By Eq. 12b, when $Q_t = 1.000$,

$$v_1 = \frac{1}{a}\frac{d\kappa}{dz} \quad \text{(17)}$$

Values of $\frac{d\kappa}{dz}$ were scaled from a curve similar to Fig. 6. A graph of v_1 is shown in Fig. 7. Values for the graphs of G_1 and U_1 in Fig. 7 were obtained by plotting values of

$$\frac{dG_1}{dz} = \frac{\rho}{a}\left(\frac{d\kappa}{dz}\right)^2 \qquad (18a)$$

and

$$\frac{dU_1}{dz} = z\,\frac{dG_1}{dz} \qquad (18b)$$

against values of z and measuring the areas under the curves by means of a planimeter. Values of Q_1 are represented by the ordinates in Fig. 6, and, using

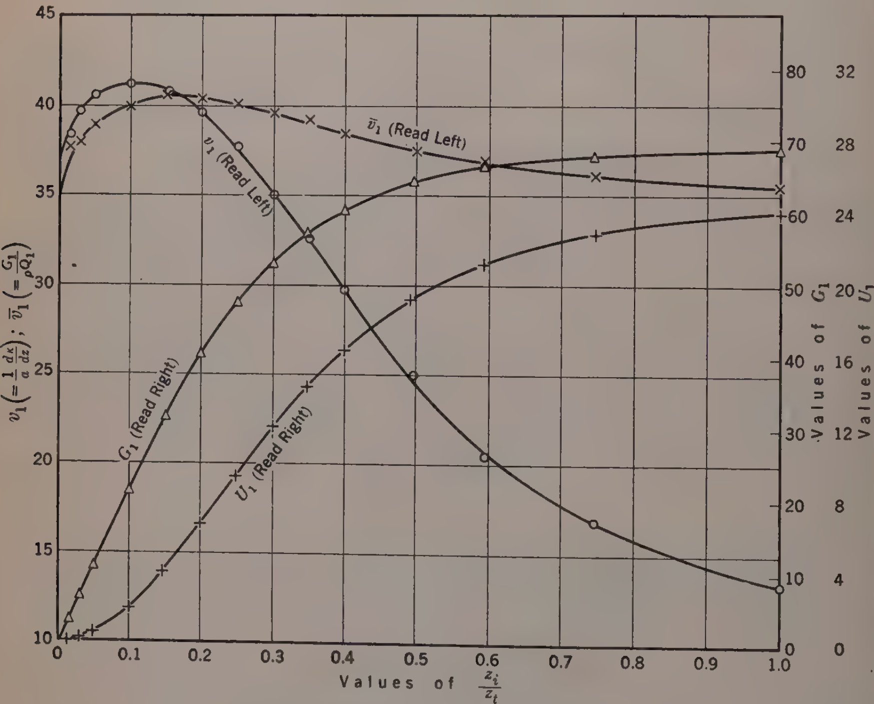

FIG. 7.—DISTRIBUTION DATA FOR $a_0 = 0.0460$, $Q_t = 1.000$

the values of G_1 from Fig. 7, values of $\bar{v}_1$ are obtainable by dividing G_1 by $\rho\, Q_1$, and are plotted in Fig. 7. Values of $\bar{z}\,P$ in Fig. 8 were likewise obtained by measuring the areas under the graph of $z\,\frac{dP}{dz}$ as a function of z.

From the experimental values of $\frac{z_i}{z_t}$ in Table 1 and the graphs in Figs. 5 to 8, the other necessary gravitational data for beginning analysis can be obtained, and these are shown in Table 1.

V.—ANALYSIS

Form of the Function of **F**.—Since the phenomenon of flow into an inlet is principally gravitational in character, and since the flow is turbulent rather

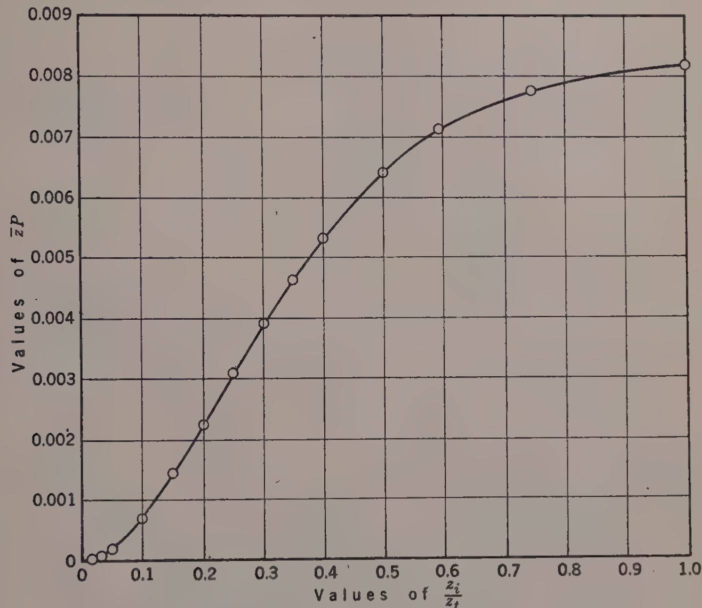

FIG. 8.—DISTRIBUTION OF MOMENT OF PRESSURE (a_0 = 0.0460)

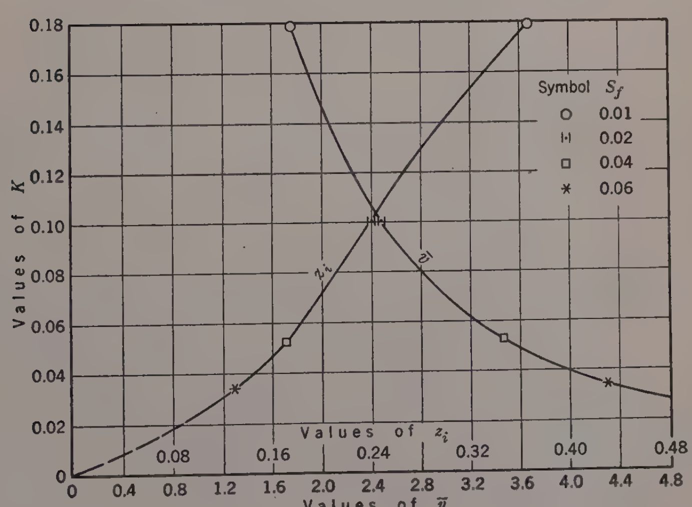

FIG. 9.—CORRESPONDING VALUES OF K, $\bar{v}$, AND z_i FOR a_0 = 0.0460 AND d = 0.0313

than laminar, K, in Eq. 6*b*, should vary principally with **F**, and to a minor extent with **R**. The independent variable K is shown as a function of $\bar{v}$ and

z_i in Fig. 9. Various flow relationships are shown in Fig. 10 and, in Fig. 11, K is shown as a function of $\mathbf{F}$, and of $\left(\frac{1}{\mathbf{F}}\right)^{0.5} \frac{1}{\mathbf{F}}, \left(\frac{1}{\mathbf{F}}\right)^{1.5}$, and $\left(\frac{1}{\mathbf{F}}\right)^{2}$.

It is evident from Fig. 11 that K can be represented nearly by a straight-line function of $\frac{1}{\mathbf{F}}$. (Since $\mathbf{F}$ represents the principal force relationship only simple exponents of $\frac{1}{\mathbf{F}}$ are rational, because the exponents of velocities involved must represent physical quantities such as energies, momentums, etc.)[4] More-

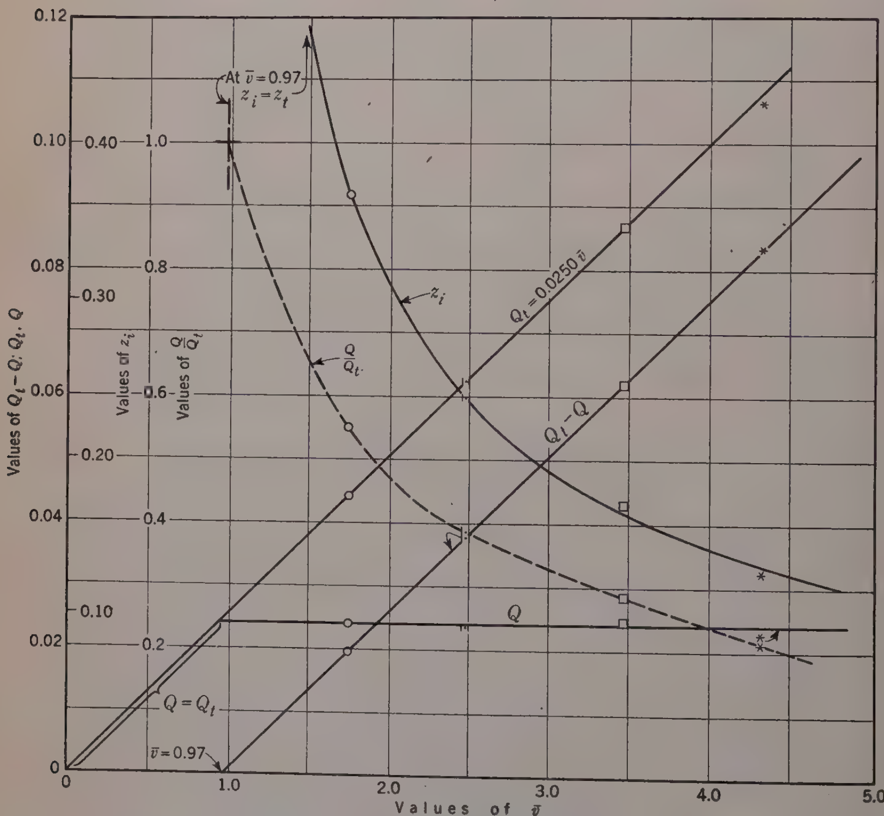

FIG. 10.—VARIOUS FLOW RELATIONSHIPS (a_0 = 0.0460; AND d = 0.0313)

over, the graph passes through the origin so that in the region of high velocities an equation of the form

$$K = \frac{\text{constant}}{\mathbf{F}} \times \phi(\mathbf{R} \ldots) = \text{constant} \times \frac{g\,h}{\bar{v}^2}\,\phi(\mathbf{R} \ldots) \ldots \ldots \ldots (19)$$

is correct. This form is also correct when g varies, because the left-hand

[4] "Engineering Mathematics," by Charles P. Steinmetz, McGraw-Hill Book Co., Inc., 1917, p. 213.

member is

$$K = \frac{\bar{z}\, P}{U} = \frac{0.5\, \rho\, g \displaystyle\int z\, a^2\, dz}{U} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

In the region of low velocities, for an inviscid fluid, the right-hand member of Eq. 19 approaches infinity. By Eqs. 9*f* and 16*a*, the left-hand member is the distributional expression

$$K = \frac{\bar{z}\, P}{U} = \frac{\dfrac{\gamma}{2} \displaystyle\int_0^{z_i} z\, a^2\, dz}{\rho\, Q^2_t \displaystyle\int_0^{z_i} \frac{z}{a} \left(\frac{d\kappa}{dz}\right)^2 dz} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

When $\bar{v}$ approaches zero, Q_t (the flow in the approach section between the curb and the crown) approaches zero. As the velocity, $\bar{v}$, of flow entering the inlet decreases, the corresponding value of z_i increases. If the maximum value of z_i is finite (as in actual fact it must be), then the denominator of Eq. 21 ap-

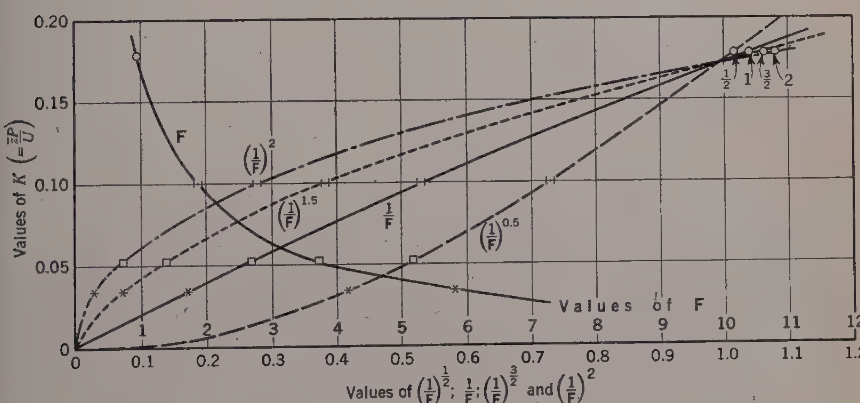

Fig. 11.—*K* as a Function of Various Powers of **F** (a_0 = 0.0460; and d = 0.0313)

proaches zero while the numerator is finite. In this case, K, the left-hand member of Eq. 19, also approaches infinity so that the form of this equation is again satisfactory.

Theoretically, z_i can increase to values greater than those of z_t if the crown board is removed and the channel is extended laterally at crown elevation. Since the depth in the extended part is constant ($= a_c$), the velocity will become practically constant and equal to v_c a short distance beyond the crown. The values of the integrals in Eq. 21 then become infinite so that $K = \dfrac{\infty}{0 \times \infty}$. This indeterminate quantity can be evaluated by differentiating under the integral signs. (Each of the integrals is a function of z_i, its upper limit, considered as a variable. Moreover, z can be substituted for z_i so that,

for example,[5] $\int_0^{z_i} z\, a^2\, dz = f(z)$, and $\frac{df(z)}{dz} = z\, a^2$.) Eq. 21 then becomes

$$K = \frac{\frac{\gamma}{2} z\, a^2}{\rho\, Q^2{}_t \frac{z}{a}\left(\frac{d\kappa}{dz}\right)^2} = \infty \quad \ldots\ldots\ldots\ldots (22)$$

when $Q_t = 0$. In either case, therefore, K becomes infinite as $\bar{v}$ approaches zero $\left(\frac{1}{\mathbf{F}}\text{ approaches infinity}\right)$; for the time being it can be assumed that the form of function of $\mathbf{F}$ in Eq. 19 is correct.

Form of Function of **R**.—The form of function of **R** can now be studied by investigating the variation of $K\,\mathbf{F}$ with respect to **R**, since, by Eq. 19,

$$K\,\mathbf{F} = f(\mathbf{R}) \quad \ldots\ldots\ldots\ldots (23)$$

It can be shown at once that the function of **R** is not of power form, such as

$$f(\mathbf{R}) = C\,\mathbf{R}^n = C\left(\frac{\bar{v}\,h}{\nu}\right)^n \quad \ldots\ldots\ldots\ldots (24)$$

because, in this form, when $\nu = 0$, as in the "perfect fluid,"

$$K = \frac{C}{\mathbf{F}}\left(\frac{\bar{v}\,h}{\nu}\right)^n = \infty \quad \ldots\ldots\ldots\ldots (25)$$

for any positive value of n—a manifest incongruity. Negative values of n yield the equally absurd relationship $K = 0$ when $\nu = 0$.

Further investigation of the form of the function of **R** will be facilitated by an analytical investigation of the limiting values of the distributional expression for $K\,\mathbf{F}$. This expression is

$$K\,\mathbf{F} = \frac{\frac{\gamma}{2}\int_0^{z_i} z\, a^2\, dz\left(\frac{\int_0^{z_i} v^2\, a\, dz}{\int_0^{z_i} v\, a\, dz}\right)^2}{\rho\, g\, h\int_0^{z_i} z\, v^2\, a\, dz} = \frac{1}{2}\,\frac{\int_0^{z_i} z\, a^2\, dz\left[\frac{\int_0^{z_i}\frac{1}{a}\left(\frac{d\kappa}{dz}\right)^2 dz}{\int_0^{z_i}\frac{d\kappa}{dz}\, dz}\right]^2}{h\int_0^{z_i}\frac{z}{a}\left(\frac{d\kappa}{dz}\right)^2 dz} \quad \ldots (26)$$

canceling Q_t. Hence $K\,\mathbf{F}$ is independent of Q_t, a fact also confirmed by experimental data. In Eq. 26, if v is uniform,

$$K\,\mathbf{F} = \frac{\int_0^{z_i} z\, a^2\, dz}{2\, h\int_0^{z_i} z\, a\, dz} = \frac{\bar{a}}{2\, h} \quad \ldots\ldots\ldots\ldots (27)$$

[5] "Advanced Calculus," by Edwin Bidwell Wilson, Ginn & Co. (Copyright 1911, 1912), p. 27.

in which $\bar{a}$ is a weighted or average value of a, and this value depends only upon the geometrical shape of the cross section and on the value of the limit z_i of the integrals. The locus of $K\,\mathbf{F}$ is thus, in this case, simply a geometrical curve. In Eq. 27, if the depth is constant ($= a$), then $K\,\mathbf{F}$ has the constant value $\frac{a}{2\,h}$ for all values of z_i.

On the other hand, when v varies with z, $K\,\mathbf{F}$ is a function of v. Moreover, in Eq. 26, z_i decreases as the approach velocity increases, and when z_i approaches zero $K\,\mathbf{F}$ takes on the indeterminate form: $K\,\mathbf{F} = \frac{0}{0} \times \frac{0}{0}$. Resorting, as before, to differentiation, and substituting for a the quantity a_0, which it approaches in value,

$$K\,\mathbf{F} = \frac{\dfrac{z\,a^2}{2}\left[\dfrac{\dfrac{1}{a}\left(\dfrac{d\kappa}{dz}\right)^2}{\dfrac{d\kappa}{dz}}\right]^2}{h\,\dfrac{z}{a}\left(\dfrac{d\kappa}{dz}\right)^2} = \frac{a_0}{2\,h} = \frac{A_0}{2} \qquad (28)$$

when $\mathbf{R}$ approaches infinity.

In like manner, when $\bar{v}$ approaches zero, or z_i approaches infinity, a approaches a_c, and $K\,\mathbf{F}$ takes on the indeterminate form: $K\,\mathbf{F} = \frac{\infty}{\infty} \times \frac{\infty}{\infty}$. Again, by differentiation and substitution, when $\mathbf{R} = 0$,

$$K\,\mathbf{F} = \frac{a_c}{2\,h} = \frac{A_c}{2} \qquad (29)$$

Eqs. 28 and 29 furnish valuable additional data for supplementing the limited range of experimental values.

It is noteworthy that in the theoretical case of uniform velocity throughout the cross section and in the two cases representing limiting values of approach velocity K takes on the form of a Froude number: By Eq. 27, with v uniform—

$$K = \frac{\bar{a}}{2\,h} \times \frac{g\,h}{\bar{v}^2} = \frac{1}{2}\,\frac{\bar{a}\,g}{\bar{v}^2} \qquad (30a)$$

by Eq. 28, with $z_i = 0$—

$$K = \frac{a_0}{2\,h} \times \frac{g\,h}{\bar{v}^2} = \frac{1}{2}\,\frac{a_0\,g}{\bar{v}^2} \qquad (30b)$$

and, by Eq. 29, with $z_i = \infty$—

$$K = \frac{a_c}{2\,h} \times \frac{g\,h}{\bar{v}^2} = \frac{1}{2}\,\frac{a_c\,g}{\bar{v}^2} \qquad (30c)$$

Derivation of the Equation.—For the experimental case being investigated $a_0 = 0.0460$, $a_c = 0.0035$, and $h = 0.1001$. Substituting these values in Eq. 28 and Eq. 29, there results: For $z_i = 0$,

$$K_0\,\mathbf{F}_0 = \frac{0.0460}{2 \times 0.1001} = 0.230 \qquad (31a)$$

and, for $z_i = \infty$,

$$K_c \mathbf{F}_c = \frac{0.0035}{2 \times 0.1001} = 0.0175 \qquad (31b)$$

Moreover, it was observed in developing distribution curves that $K\,\mathbf{F}$ has the constant value $K_t\,\mathbf{F}_t = 0.133$ for all values of Q_t. These values are plotted, together with the four experimental ones, in Fig. 12. From the shape of the

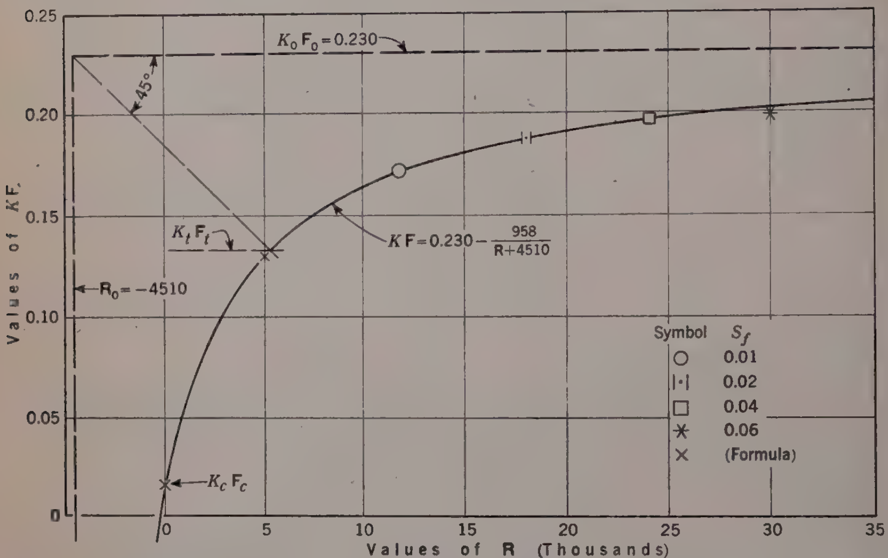

FIG. 12.—$K\,\mathbf{F}$ AS A FUNCTION OF $\mathbf{R}$ ($a_0 = 0.0460$; AND $d = 0.0313$)

curve and the physical fact that $K\,\mathbf{F}$ must approach a constant value asymptotically when $\mathbf{R}$ approaches infinity, it is suggested that an hyperbola should fit the data. It may be a mere coincidence that the value of $K_t\,\mathbf{F}_t$ falls on the graph at a point on a line which bisects the angle between the asymptotes.

The three-constant form of the equation for an hyperbola can be written

$$(\mathbf{R} - \mathbf{R}_0)\,(K\,\mathbf{F} - K_0\,\mathbf{F}_0) = -\,C_0 \qquad (32a)$$

in which $\mathbf{R}_0$ and $K_0\,\mathbf{F}_0$ are the asymptotes and C_0 is a constant of the hyperbola, or

$$K\,\mathbf{F} = 0.5\,A_0 - \frac{C_0}{\mathbf{R} - \mathbf{R}_0} \qquad (32b)$$

This is in the form of Eq. 23.

By Eqs. 29 and 32*b*, when $\mathbf{R} = 0$,

$$\frac{C_0}{\mathbf{R}_0} = -\frac{a_0 - a_c}{2\,h} = -\frac{y_c}{2\,h} = -\,0.5\,Y_c \qquad (33)$$

in which y_c is the crown height.

Only one constant, $\mathbf{R}_0$, remains to be determined; any one of the experimental sets of data is sufficient for its evaluation. Computation of a value

corresponding to each set will provide comparative data by means of which the reliability of the method and the accuracy of the data can be judged. Solving for $\mathbf{R}_0$,

$$\mathbf{R}_0 = \frac{(K\,\mathbf{F} - 0.5\,A_0)\,\mathbf{R}}{K\,\mathbf{F} - 0.5\,A_c} \quad \text{(34)}$$

Values of $\mathbf{R}_0$ for $d = 0.0313$ are computed in Table 2. The values in the last row in the table represent the arithmetical average values of $K\,\mathbf{F}$ and $\mathbf{R}$ for

TABLE 2.—VALUES OF $\mathbf{R}_0$ FOR $d = 0.0313$

S_f (1)	$K\,\mathbf{F}$ (2)	$\mathbf{R}$ (3)	$K\,\mathbf{F} - 0.5\,\mathbf{A}_0$ (4)	Col. 4 × R (5)	$K\,\mathbf{F} - 0.5\,\mathbf{A}_c$ (6)	$\mathbf{R}_0$ (7)	$\mathbf{R}_0$ (aver.) − $\mathbf{R}_0$ (8)	Discrepancy (% of aver.) (9)	− 4,510 − $\mathbf{R}_0$ (10)	% of − 4,510 (11)
0.01	0.172	11,810	−0.058	−685	+0.1545	−4,430	−175	+ 3.8	− 80	+ 1.8
0.02	0.189	18,030	−0.041	−739	+0.1715	−4,310	−295	+ 6.4	−200	+ 4.4
0.04	0.196	24,040	−0.034	−818	+0.1785	−4,580	− 25	+ 0.5	+ 70	− 1.6
0.06	0.199	29,970	−0.031	−928	+0.1815	−5,100	+495	−10.8	+590	−13.1
Aver., S_f = 0.01, 0.02, 0.04, 0.06				....	....	−4,605	....	....	....	....
Aver.[a]	0.1925	21,035	−0.0375	−790	+0.1750	−4,510	[a] Averages for S_f = 0.02 and 0.04			

$S_f = 0.02$ and 0.04. These two sets are believed to be the most reliable. Hence, the value $\mathbf{R}_0 = -4{,}510$ will be considered satisfactory. By Eq. 33 $\left(\text{since } Y_c = \frac{0.0425}{0.1001} = 0.425\right)$, $C_0 = -0.5 \times 0.425\,(-4{,}510) = +958$. The equation representing $a_0 = 0.0460$, $d = 0.0313$ is then, by Eq. 32*b*:

$$K\,\mathbf{F} = 0.230 - \frac{958}{\mathbf{R} + 4{,}510} \quad \text{(35)}$$

The check computations in Table 3 are obtained by inserting experimental

TABLE 3.—COMPUTATIONS TO CHECK EQ. 35

S_f	$K\,\mathbf{F}$	Discrepancy	% of observed $K\,\mathbf{F}$	% of observed range
0.01	$0.230 - \frac{958}{11{,}810 + 4{,}510} = 0.1713$	− 0.0007	− 0.40	− 2.59
0.02	$0.230 - \frac{958}{18{,}030 + 4{,}510} = 0.1875$	− 0.0015	− 0.79	− 5.56
0.04	$0.230 - \frac{958}{24{,}040 + 4{,}510} = 0.1965$	+ 0.0005	+ 0.25	+ 1.85
0.06	$0.230 - \frac{958}{29{,}970 + 4{,}510} = 0.2022$	+ 0.0032	+ 1.61	+ 11.85
($\mathbf{R}$ = 5,000)	$0.230 - \frac{958}{5{,}000 + 4{,}510} = 0.1293$	....	....	....

values of $\mathbf{R}$ in Eq. 35. Thus, in all cases the values of $K\,\mathbf{F}$ computed from Eq. 35 are within 1.7% of the observed values of $K\,\mathbf{F}$. Comparison of the discrepancies on a basis of percentage of the small observed range (= 0.027)

of values of K **F** yields less than 6% for the first three slopes and less than 12% for $S_f = 0.06$. In the last case the value of z_i represents a distance less than that between the curb and the first vane by which the flow distribution was investigated, and a region, moreover, where the velocity profile curves rapidly. The data for $S_f = 0.06$, therefore, should be accredited less weight than the others. The results, in general, are considered satisfactory.

The equation for K is then

$$K = \frac{1}{\mathbf{F}}\left(0.230 - \frac{958}{\mathbf{R} + 4{,}510}\right) \qquad (36a)$$

or, in general terms,

$$K = \frac{1}{\mathbf{F}}\left[\frac{a_0}{2\,h} + \frac{\dfrac{a_0 - a_c}{2\,h}\mathbf{R}_0}{\mathbf{R} - \mathbf{R}_0}\right] = \frac{0.5}{\mathbf{F}}\left(A_0 + \frac{Y_c\,\mathbf{R}_0}{\mathbf{R} - \mathbf{R}_0}\right) \qquad (36b)$$

The Function When **D** *Varies.*—It can now finally be demonstrated that the form of function of **F** in Eq. 19 is consistent when **D** (and, therefore, h) varies, because, by Eq. 36b,

$$K = \frac{a_0\,g}{2\,\bar{v}^2}\left(1 + \frac{a_0 - a_c}{a_0}\,\frac{\mathbf{R}_0}{\mathbf{R} - \mathbf{R}_0}\right) \qquad (37)$$

but,

$$\frac{\mathbf{R}_0}{\mathbf{R} - \mathbf{R}_0} = \frac{\mathbf{R}_0}{\mathbf{R}} + \left(\frac{\mathbf{R}_0}{\mathbf{R}}\right)^2 + \left(\frac{\mathbf{R}_0}{\mathbf{R}}\right)^3 + \cdots \qquad (38a)$$

in which each term $\frac{\mathbf{R}_0}{\mathbf{R}}$ can be written

$$\frac{\mathbf{R}_0}{\mathbf{R}} = \frac{\dfrac{\bar{v}_0\,h}{\nu}}{\dfrac{\bar{v}\,h}{\nu}} = \frac{\bar{v}_0}{\bar{v}} \qquad (38b)$$

Hence, when ν is the same for different values of h,

$$K = \frac{a_0\,g}{2\,\bar{v}^2}\left(1 + \frac{a_0 - a_c}{a_0}\,\frac{\bar{v}_0}{\bar{v} - \bar{v}_0}\right) \qquad (39)$$

In Eq. 39, h does not appear in the right-hand member, and since, by Eq. 21, it is also absent from the left-hand member, it must be concluded that the form (Eq. 19) is correct. The small variation of K with h, which can be noted in Table 1, is in conformity with the small variation of $\bar{v}$ with z_i when Q_t is constant as shown in Fig. 7.

Shape Parameters.—In Eq. 36b, A_0, Y_c, and $\mathbf{R}_0$ are parameters whose values depend upon shape factors. The physical meaning of this equation is made more apparent when it is noted that, for high values of **R** (high velocities and small widths of inlet flow), K depends principally upon A_0; whereas, for lower values of **R** (lower velocities and wider filaments of inlet flow), K depends to an increased extent upon Y_c and $\mathbf{R}_0$.

The values of A_0 and Y_c depend upon the shape of the approach flow (if d is constant). When that shape is constant, other form factors such as length of inlet, width of depression, etc., govern the value of $\mathbf{R}_0$. Evidently K will be especially affected by variations of the latter form factors when approach velocities are low.

TABLE 4.—VALUES OF $\mathbf{R}_0$

d	h	D	$0.5\ A_0$	$0.5\ A_c$	$K\mathbf{F}$	$\mathbf{R}$	$K\mathbf{F} - 0.5\ A_0$	$(K\mathbf{F} - 0.5\ A_0)\mathbf{R}$	$K\mathbf{F} - 0.5\ A_c$	$\mathbf{R}_0$
0.0000	0.0688	0.000	0.334	0.0254	0.274	13,070	−0.060	− 783	0.249	−3,140
0.0625	0.1313	0.476	0.175	0.0133	0.128	24,150	−0.047	−1,135	0.115	−9,870

Values of $\mathbf{R}_0$ *for* $D = 0.000$ *and* $D = 0.476$.—The values of $\mathbf{R}_0$ in Table 4 were obtained by substituting experimental data in Eq. 34. Various relationships between $\mathbf{R}_0$, D, d, and $f(D)$ are shown in Appendix A.

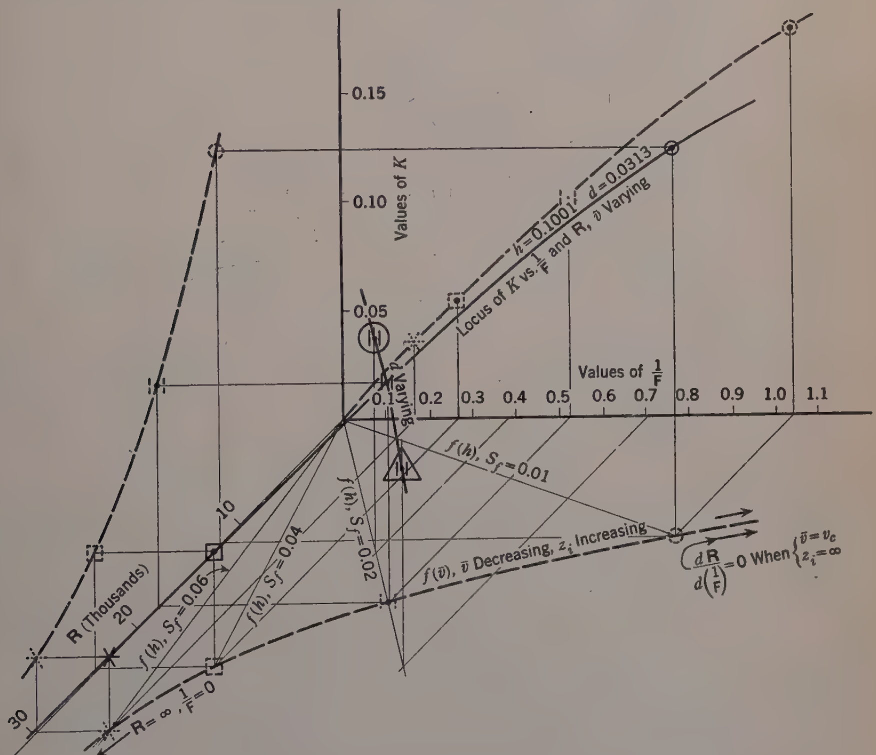

FIG. 13.—EXPERIMENTAL LOCUS OF K AS A FUNCTION OF $\mathbf{F}$ AND $\mathbf{R}$

VI.—APPLICATION TO DESIGN

Eqs. 36 describe the experimental locus of K as a function of $\mathbf{F}$ and $\mathbf{R}$. The locus is shown in three coordinates in Fig. 13. It is now desired to develop an

expression by means of which the model results can be applied to a full-size prototype. This requires a separation of the effects of **F** and **R** so that a value of **R** can be used which will represent viscous conditions similar to those of a prototype. For model size and velocities this involves substituting a smaller value of ν.

Experimental values of K are plotted as a function of $\frac{1}{\mathbf{F}}$ (solid graph) in Fig. 14. It is evident that the values of K in this figure represent the combined effect of **F** and **R**. This follows from the fact that **F** and **R** vary simultaneously

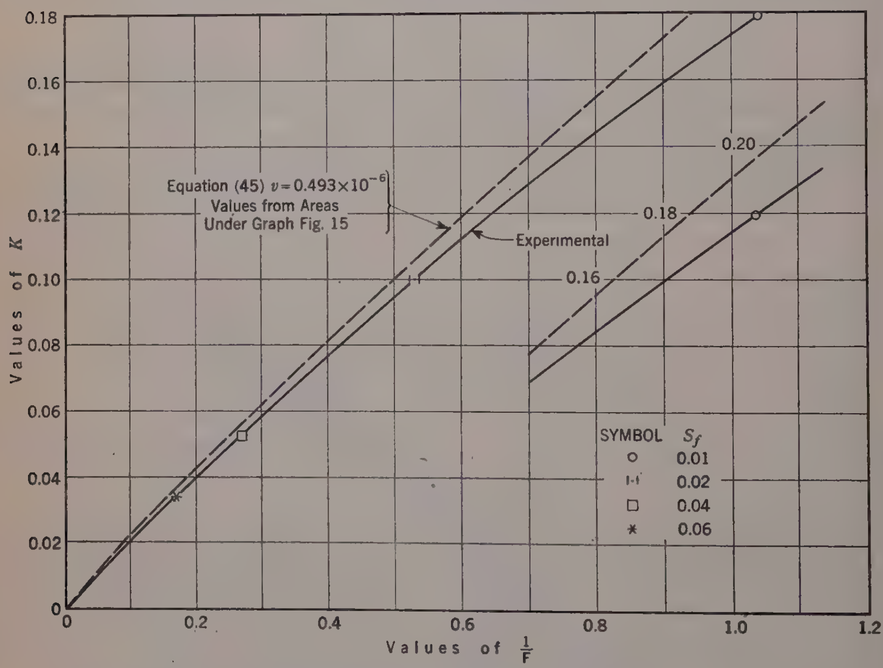

FIG. 14.—EXPERIMENTAL AND THEORETICAL LOCI OF K AS A FUNCTION OF 1/**F** (a_0 = 0.0460; AND d = 0.0313)

when $\bar{v}$ varies. The slope of the graph, as plotted, represents the total derivative of K with respect to $\frac{1}{\mathbf{F}}$, because K, then, is a function of the two independent variables **F** and **R**, both of which can be expressed in terms of **F**. The method pursued in the following involves obtaining an expression for the total derivative, inserting a value of ν lower than the experimental value and then integrating. The result is shown as the broken graph in Fig. 14.

An expression for the total derivative is:

$$\frac{dK}{d(1/\mathbf{F})} = \frac{\partial K}{\partial(1/\mathbf{F})} + \frac{\partial K}{\partial \mathbf{R}} \frac{d\mathbf{R}}{d(1/\mathbf{F})} \qquad (40a)$$

From Eq. 36*b*,

$$\frac{\partial K}{\partial(1/\mathbf{F})} = \frac{a_0}{2\,h} + \frac{\dfrac{a_0 - a_c}{2\,h}}{\mathbf{R}_M - \mathbf{R}_0}\mathbf{R}_0 \quad\text{.....}(40b)$$

Note by comparison with Eq. 36*b* that this equals $K\,\mathbf{F}$. The subscript M in Eq. 40*b* indicates the constancy of $\mathbf{R}$ ($\bar{v}$, h, and ν must have the model values —see Fig. 14—at the point concerned). Moreover,

$$\frac{\partial K}{\partial \mathbf{R}} = -\frac{1}{\mathbf{F}_M}\left[\frac{\dfrac{a_0 - a_c}{2\,h}\mathbf{R}_0}{(\mathbf{R}_\nu - \mathbf{R}_0)^2}\right] \quad\text{.....}(41)$$

the subscript ν indicating that $\mathbf{R}_\nu$ is arbitrary with regard to ν, but in other respects it must be consistent with the constancy in $\mathbf{F}$.

Reynolds' number may be expressed in terms of $\frac{1}{\mathbf{F}}$ in the following manner:

$$\bar{v}^2 = \frac{g\,h}{1/\mathbf{F}} \quad\text{.....}(42a)$$

and

$$\mathbf{R} = \left(\frac{g\,h}{1/\mathbf{F}}\right)^{0.5}\frac{h}{\nu} \quad\text{.....}(42b)$$

Thus

$$\frac{d\mathbf{R}}{d(1/\mathbf{F})} = \frac{g^{0.5}\,h^{1.5}}{\nu}\left[-\frac{1}{2}\left(\frac{1}{\mathbf{F}}\right)^{-1.5}\right] \quad\text{.....}(43a)$$

Eq. 43*a* can be written

$$\frac{d\mathbf{R}}{d(1/\mathbf{F})} = -\frac{1}{2}\frac{\bar{v}\,h}{\nu} \times \frac{\bar{v}^2}{g\,h} = -\frac{1}{2}\mathbf{F}_M\,\mathbf{R}_\nu \quad\text{.....}(43b)$$

Then inserting Eqs. 40*b*, 41, and 43*b* in Eq. 40*a*, the total derivative ($\bar{v}$ varying), is

$$\frac{dK}{d(1/\mathbf{F})} = \frac{a_0}{2\,h} + \frac{\dfrac{a_0 - a_c}{2\,h}\mathbf{R}_0}{\mathbf{R}_M - \mathbf{R}_0} + \frac{1}{2}\frac{\dfrac{a_0 - a_c}{2\,h}\mathbf{R}_0}{(\mathbf{R}_\nu - \mathbf{R}_0)^2}\mathbf{R}_\nu = \mathrm{A} + \mathrm{B} + \mathrm{C} \quad\text{....}(44)$$

If the value of $\mathbf{R}_\nu$ becomes infinite (ν approaches zero), the last term (C) approaches zero in value, and the sum of the other two (A + B) represents the slope of the graph for an inviscid fluid. Moreover, the sum of the two terms equals the partial derivative as given in Eq. 40*b*, which, as before stated, equals the value of $K\,\mathbf{F}$ as given by Eq. 36*b*.

With regard to the effects of viscosity, flow in the model is similar to flow in a prototype when Reynolds' number is the same in both systems. Eq. 44 thus may be made to represent a prototype by inserting the proper values of ν in the terms $\mathbf{R}_\nu$, but using experimental values of $\bar{v}$. Values of K are then given by

$$K = \int \frac{dK}{d(1/\mathbf{F})}\,d\left(\frac{1}{\mathbf{F}}\right) = \int\left[\frac{a_0}{2\,h} + \frac{\dfrac{a_0 - a_c}{2\,h}\mathbf{R}_0}{\mathbf{R}_M - \mathbf{R}_0} + \frac{1}{2}\frac{\dfrac{a_0 - a_c}{2\,h}\mathbf{R}_0\,\mathbf{R}_\nu}{(\mathbf{R}_\nu - \mathbf{R}_0)^2}\right]d\left(\frac{1}{\mathbf{F}}\right) \quad\text{..}(45)$$

The integral can be evaluated by measuring the area under the graph of the integrand, $\frac{dK}{d(1/\mathbf{F})}$, as a function of $\frac{1}{\mathbf{F}}$.

To obtain a value of ν, the case representing $S_f = 0.02$ is taken as an average one. For this case, $\mathbf{R} = 18{,}030$. Let it be assumed that the value of **R** for a prototype is about 500,000. (The exact value is not important, because the rate of variation of K with **R** in the region of $\mathbf{R} = 500{,}000$ is very small; $\mathbf{R}_\nu$ could be considered infinite without great error. The last term in Eq. 45 represents viscous effect.) The total flow, by Chezy's formula, in a prototype channel whose dimensions are $\frac{32}{3}$ times those of the model ($a_0 = 0.0460$), and for which Kutter's coefficient of roughness is $n = 0.013$, is Q_{tP} = area $\times\ C_P\ (R_P\ S)^{0.5} = 3.40 \times 79\ (0.184 \times 0.02)^{0.5} = 16.3$.

Using subscripts M and ν to denote model and prototype (both to be simulated with model size, however), the relationship between Reynolds' numbers and values of ν (h and $\bar{v}$ being constant) is given by

$$\frac{\mathbf{R}_M}{\mathbf{R}_\nu} = \frac{\frac{\bar{v}\,h}{\nu_M}}{\frac{\bar{v}\,h}{\nu_\nu}} = \frac{\nu_\nu}{\nu_M} \quad \text{.......................(46)}$$

Since $\nu_\nu = \nu_M \frac{\mathbf{R}_M}{\mathbf{R}_\nu}$, $\nu_\nu = 13.7 \times 10^{-6} \times \frac{18{,}030}{500{,}000} = 0.493 \times 10^{-6}$.

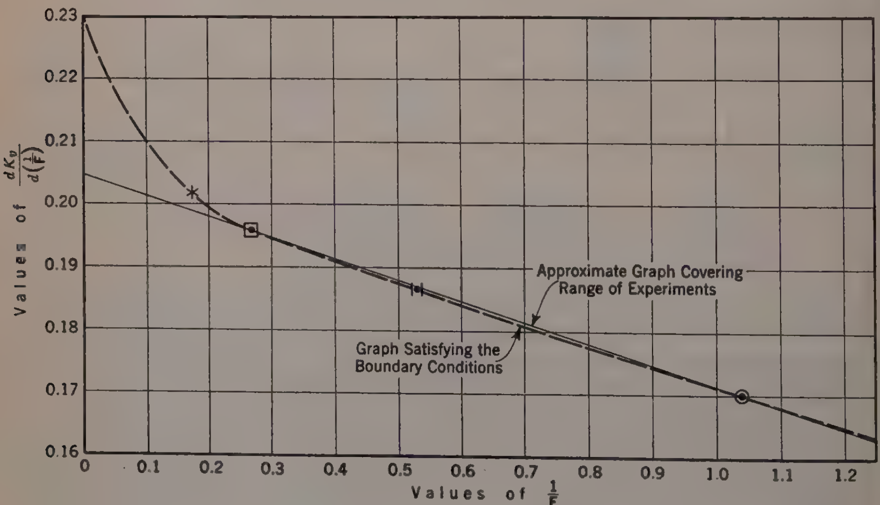

FIG. 15.—VALUES OF $\frac{dK_\nu}{d(1/\mathbf{F})}$ CORRESPONDING TO $\nu = 0.493 \times 10^{-6}$ ($a_0 = 0.0460$)

Values of $\frac{dK_\nu}{d(1/\mathbf{F})}$ corresponding to the value of $\nu = 0.493 \times 10^{-6}$ are calculated in Table 5 and have been plotted in Fig. 15 for the four experimental

cases and the theoretical limiting case, $\frac{1}{\mathbf{F}} = 0$; that is, $\frac{dK}{d(1/\mathbf{F})} = \frac{a_0}{2\,h}$ ($= 0.230$). Fig. 15 also holds for $\nu = 0.322 \times 10^{-6}$ (see Table 5).

TABLE 5.—CALCULATION OF VALUES OF $\frac{dK_\nu}{d(1/\mathbf{F})}$

$$\left(\frac{a_0}{2\,h} = 0.230;\ \frac{a_0 - a_c}{2\,h}\,\mathbf{R}_0 = -958;\ \text{and}\ \mathbf{R}_0 = -4{,}510\right)$$

S_f	$\mathbf{R}_M$	$\frac{1}{\mathbf{F}_M}$	$\bar{v}$	$\mathbf{R}_\nu$ $\left(=\frac{\bar{v}\,h}{\nu_\nu}\right)$	$\mathbf{R}_\nu - \mathbf{R}_0$	$(\mathbf{R}_\nu - \mathbf{R}_0)^2$	$\mathbf{R}_M - \mathbf{R}_0$	Quantity B, Eq. 44	Quantity C, Eq. 44	$\frac{dK_\nu}{d(1/\mathbf{F})}$ (Eq. 44)	K_ν (Fig. 14)	$K_\nu\,\mathbf{F}$
						$\nu_\nu = 0.493 \times 10^{-6}$						
0.01[a]	11,810	1.038	1.761	0.357[b]	0.362[b]	131[c]	16,320	−0.0587	−0.00131	0.1700	0.196	0.189
0.02	18,030	0.5285	2.468	0.501[b]	0.506[b]	256[c]	22,540	−0.0424	−0.00094	0.187	0.106	0.200
0.04	24,040	0.2688	3.461	0.702[b]	0.707[b]	498[c]	28,550	−0.0335	−0.00068	0.196	0.0562	0.209
0.06	29,970	0.172	4.31	0.873[b]	0.878[b]	770[c]	34,480	−0.0278	−0.00054	0.202	0.0367	0.213
						$\nu_\nu = 0.392 \times 10^{-6}$						
0.01	11,810	1.038	1.761	0.449[b]	0.454[b]	206[c]	16,320	−0.0587	−0.00104	0.170	....	....
0.02	18,030	0.5285	2.468	0.629[b]	0.634[b]	402[c]	22,540	−0.0424	−0.00075	0.187	....	....
0.04	24,040	0.2688	3.461	0.883[b]	0.888[b]	788[c]	28,550	−0.0335	−0.00054	0.196	....	....
0.06	29,970	0.172	4.31	1.098[b]	1.103[b]	1,220[c]	34,480	−0.0278	−0.00043	0.202	....	....

[a] For $S_f = 0.01$, furthermore, $\frac{z_{i\nu}}{z_{t\nu}}$ (Fig. 16) $= 0.140$; and $\frac{Q_\nu}{Q_{t\nu}} = 0.385$. [b] Multiply by 1,000,000. [c] Multiply by 1,000,000,000.

For comparison, values of $\frac{dK_\nu}{d(1/\mathbf{F})}$ are also shown in Table 6 together with experimental values of $K\,\mathbf{F}$ and calculated values of $\frac{\partial K}{\partial(1/\mathbf{F})}$ (Eq. 40*b*), of

TABLE 6.—COMPARISON OF VALUES OF $\frac{dK_\nu}{d(1/\mathbf{F})}$

S_f	$\frac{dK_\nu}{d(1/\mathbf{F})}$ ($\nu = 0.493 \times 10^{-6}$)	$K\,\mathbf{F}$ (Observed)	$\frac{\partial K}{\partial(1/\mathbf{F})}$ (Eq. 40*b*)	$\frac{dK_M}{d(1/\mathbf{F}_M)}$ (Eq. 44)	$\frac{a_i}{2\,h}$	K (Eq. 36*b*)	K (Observed)	K_ν (Fig. 14)	$\frac{a_i}{2\,h} \times \frac{1}{\mathbf{F}}$
0.01	0.170	0.172	0.171	0.150	0.148	0.177	0.179	0.196	0.153
0.02	0.187	0.189	0.188	0.170	0.175	0.0989	0.100	0.106	0.0919
0.04	0.196	0.196	0.196	0.182	0.190	0.0527	0.0527	0.0562	0.0508
0.06	0.202	0.199	0.202	0.190	0.199	0.0348	0.0343	0.0367	0.0343

$\frac{dK_M}{d(1/\mathbf{F}_M)}$ (Eq. 44, $\nu_\nu = \nu_M$), of $\frac{a_i}{2\,h}$, and of $\frac{a_i}{2\,h} \times \frac{1}{\mathbf{F}}$. $\Bigl($Note that $\frac{a_i}{2\,h} \times \frac{1}{\mathbf{F}} = \frac{1}{2\,\bar{v}^2/(a_i\,g)}$, in which $\frac{\bar{v}^2}{a_i\,g}$ is a Froude number.$\Bigr)$ It may be noted that throughout the experimental range the corresponding values of $\frac{dK_\nu}{d(1/\mathbf{F})}$, $K\,\mathbf{F}$,

and $\frac{\partial K}{\partial(1/\mathbf{F})}$ are substantially equal and that, for $S_f = 0.06$, values of the three quantities practically equal that of $\frac{a_i}{2h}$. Values of the total derivative, $\frac{dK_M}{d(1/\mathbf{F})}$, representing the actual experimental value of ν are approximately equal to values of $\frac{a_i}{2h}$, especially for the lower values of S_f (greater values of z_i).

In Fig. 15 a straight line represents, with sufficient accuracy, the three points representing $S_f = 0.01$, 0.02, and 0.04, but passes through neither the limiting point $\frac{dK}{d(1/\mathbf{F})} = 0.230$ at $\frac{1}{\mathbf{F}} = 0$, nor the other extreme, $\frac{dK}{d(1/\mathbf{F})} = \frac{a_e}{2h}$ (= 0.0175) at $\frac{1}{\mathbf{F}} = \infty$, obtainable from Eq. 44, when $\bar{v} = 0$—that is, when $\mathbf{R}_M$ and $\mathbf{R}_\nu$ equal zero.

Values of the integral Eq. 45 have been obtained from the areas under the dashed curve, Fig. 15. These are given in Table 7.

TABLE 7.—VALUES OF THE INTEGRAL, EQ. 45

$\frac{1}{\mathbf{F}}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.9	1.1
ΔK_ν	...	0.0219	0.0204	0.0197	0.0193	0.0189	0.0186	0.0182	0.0355	0.0342
K_ν	0	0.0219	0.0423	0.0620	0.0813	0.1002	0.1188	0.1370	0.1725	0.2067

Values of K_ν (= $\Sigma \Delta K_\nu$) from Table 7 are plotted in Fig. 14 and for the four experimental points their values are shown in Table 5, and in Table 6 together with the experimental and calculated values of K. Values of $K_\nu \mathbf{F}$ have also been calculated. Fig. 16 gives the corresponding values of $\frac{z_i}{z_t}$. It may be noted that except for the data representing $S_f = 0.01$, the values of $K_\nu \mathbf{F}$ are greater than the observed distributional values of $K \mathbf{F}$. The explanation of this is that the values of the equations derived herein depend upon conditions beyond the frictional effect of the curb. Such conditions have been represented in the region of the curb by the broken line in Fig. 16. The matter is not of practical importance since corresponding prototype values of z_i are smaller than are usually encountered.

Determination of the Model Case Corresponding to a Given Prototype.—In the approach channels of model and prototype, let the ratio of corresponding flows be the same. (This is in accordance with the usual assumption of model analysis technique.) Then

$$\frac{Q_P}{Q_{tP}} = \frac{Q_M}{Q_{tM}} \qquad (47a)$$

Assume, moreover, that for corresponding inlet flows the total flows follow Froude's law, or,

$$Q_{tP} = \left(\frac{32}{3}\right)^{2.5} Q_{tM} = 372\, Q_{tM} \qquad (47b)$$

Fig. 10 then yields the value of $\bar{v}$ $\left(\text{and, hence, of } \frac{1}{\mathbf{F}}\right)$ representing the inlet flow corresponding to the prototype total flow.

Design Example.—Let it be required to determine the quantity of flow that will enter an inlet 7 ft long, with a local depression 4 in. deep. The pavement

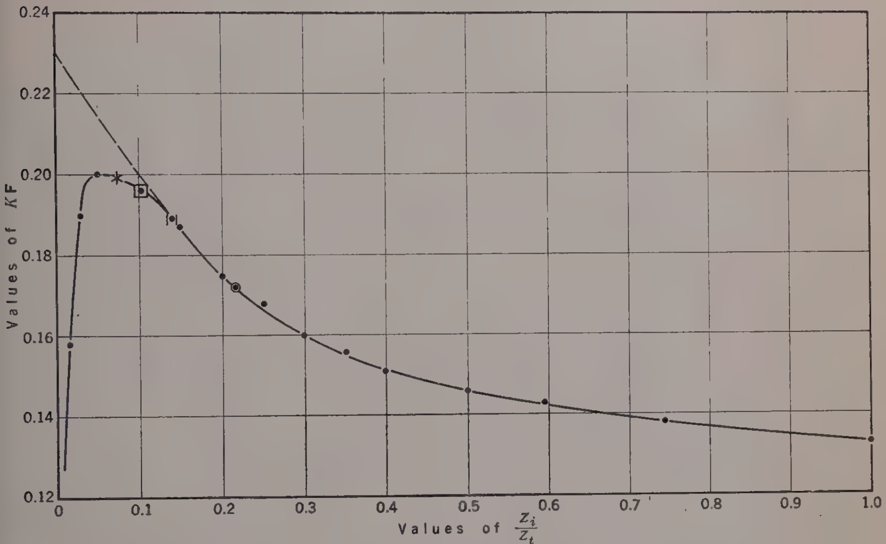

FIG. 16.—DISTRIBUTION OF $K\mathbf{F}$ FOR ALL VALUES OF Q_t (h = 0.1001)

is 36 ft wide, with a crown height of 0.45 ft and a longitudinal slope of 2%. The total flow between the curb and the crown is 16.3 cu ft per sec—the conditions corresponding to the model. (The flow in such a prototype, assuming Kutter's n = 0.013, is probably actually greater than 16.3, for the true mean depth is greater than an hydraulic radius determined by dividing the cross sectional area by the wetted perimeter.)

The design process is as follows: By Eq. 47*b*, $Q_{tM} = \frac{16.3}{372} = 0.0438$. (Note that the experimental value representing $S_f = 0.01$ is $Q_{tM} = 0.0436$.) By Fig. 10, for this amount of total flow in the model, the average velocity of the part entering the inlet is $\bar{v}_M = 1.76$; so that: $\frac{1}{\mathbf{F}_M} = \frac{32.2 \times 0.1001}{(1.76)^2} = 1.038$. The value of ν representing the prototype value of $\mathbf{R}$ with model dimensions and velocity now can be computed. Since Froude's number is the same in model and prototype, and since $h_P = 1.07$, $\bar{v}_P = \left(\frac{32.2 \times 1.07}{1.038}\right)^{0.5} = 5.76$. Hence, for $\nu_P = 13.7 \times 10^{-6}$ (the experimental value for $S_f = 0.02$), $\mathbf{R}_P = \frac{5.76 \times 1.07}{13.7 \times 10^{-6}}$ = 450,000. If Reynolds' number for model dimensions and velocities is to equal this, $\nu_\nu = \frac{1.76 \times 0.1001}{450{,}000} = 0.392 \times 10^{-6}$. Values of $\frac{dK_\nu}{d(1/\mathbf{F})}$ for this value

of ν are shown in Table 5, and, since these agree with the values representing $\nu = 0.493 \times 10^{-6}$, the corresponding graph (Fig. 14) is satisfactory for design use in the present case.

For $\frac{1}{\mathbf{F}} = 1.038$, Fig. 14 yields $K_M = 0.179$; so that $K_M \mathbf{F} = \frac{0.179}{1.038} = 0.172$; and $K_\nu = 0.196$. Therefore, $K_\nu \mathbf{F} = \frac{0.196}{1.038} = 0.189$.

From Fig. 16: $\frac{z_{iM}}{z_{tM}} = 0.217$; $\frac{z_{i\nu}}{z_{t\nu}} = 0.140$; and, by Fig. 6, $\frac{Q_\nu}{Q_{t\nu}} = 0.385$. By coincidence the latter two are the experimental values for $S_f = 0.02$. Note that geometrical similarity with respect to the location of the outer boundary of inlet flow is ignored. The ratio $\frac{Q_\nu}{Q_{t\nu}} = 0.385$ is the value representing a prototype and, except for a small correction that may be needed, will yield the desired value of Q_P.

The correction involves the fact that experimental and theoretical values of $\frac{z_i}{z_t}$ for $\frac{1}{\mathbf{F}} = 1.038$ are different, whereas the average velocity is the same. It follows that the total flow corresponding to the two cases is different. The

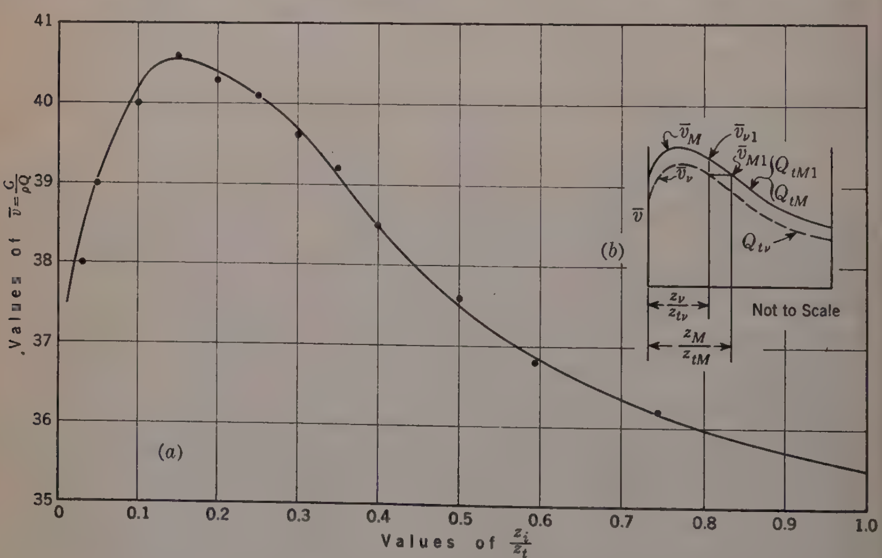

FIG. 17.—DISTRIBUTION OF $\bar{v}$ FOR UNIT TOTAL FLOW ($a_0 = 0.0460$)

relationship between the two flows is found as follows: Two values, $\bar{v}_{\nu 1}$ and $\bar{v}_{M1}$, representing unit total flow, can be obtained for the two values of $\frac{z_i}{z_t}$ from Fig. 17. The ratio of these velocities equals the ratio of two other average velocities representing two different total flows, but a given value of $\frac{z_i}{z_t}$. This

is shown schematically in Fig. 17(*b*). Hence

$$\frac{\bar{v}_{M1}}{\bar{v}_{v1}} = \frac{\bar{v}_v}{\bar{v}_M} = \frac{Q_{tv}}{Q_{tM}} \qquad (48)$$

Fig. 17(*a*) yields, for $\frac{z_{iM}}{z_{tM}} = 0.217$ and $\frac{z_{iv}}{z_{tv}} = 0.140$, $\bar{v}_{M1} = 40.28$, and $\bar{v}_{v1} = 40.55$; so that by Eq. 48 $\frac{Q_{tv}}{Q_{tM}} = \frac{40.28}{40.55} = 0.994$. Hence,

$$Q_{tv} = 0.994\, Q_{tM} \qquad (49)$$

Since $\frac{Q_v}{Q_{tv}} = 0.385$, $Q_v = 0.385 \times 0.994 \times 0.0438 = 0.0167$ (the correction is negligible in this case, and may be found to be so for all cases). The ratio $\frac{Q_v}{Q_M} = \frac{0.0167}{0.0239} = 0.699$ indicates that the flow that may be expected to enter a prototype inlet is about 70% of that which would be obtained by direct application of the model results.

Since, for equal values of **F**, $\frac{Q_P}{Q_{tP}} = \frac{Q_v}{Q_{tv}} = 0.385$, $Q_P = 16.3 \times 0.385 = 6.3$ cu ft per sec, the desired value of inlet flow in the prototype.

Where values of $\frac{Q}{Q_t}$ in the prototype are appreciably different from those in the model, it is necessary that the flow distribution be known. The variables K and $\bar{v}$ were selected with such a contingency in mind, but demonstration as to whether Froude's law applies in the case is not available at this time.

VII.—Conclusion

The result, $Q_P = 6.3$ cu ft per sec, obtained in the preceding example, is a reasonable quantity on a basis of experience, and in the absence of an error in the analytical method must be assumed to be correct. Nevertheless, confirmation of the result by means of an exactly similar larger, or full-scale, model is desirable.

The writer is aware that the method of solution used is not the only one, and, quite possibly, not the simplest, nor the best. Further investigation may disclose, for example, that the maximum velocity can be used as the characteristic velocity instead of $\bar{v}$. This would have the advantage of removing any doubt as to whether $\bar{v}$ is actually entirely independent of K. The maximum velocity can easily be obtained from Fig. 7, and it has the value

$$v_{\max} = 41.2\, Q_t \qquad (50)$$

The method of solution presented in this paper is intended to be used in future development of data for purposes of design. The necessary experimental data covering a fairly wide range of lengths of inlet, slopes of street, etc., have already been obtained.

Acknowledgments.—The experimental work upon which this paper is based was completed at the California Institute of Technology in 1937. The work was initiated and conducted mainly by the Bureau of Engineering, City of Los Angeles, through L. W. Armstrong, M. Am. Soc. C. E., the writer being in direct charge. The County of Los Angeles, through C. E. Arnold, Assoc. M.

Am. Soc. C. E., cooperated by furnishing an assistant. Valuable criticisms and suggestions have been contributed by R. T. Knapp, M. Am. Soc. C. E., V. A. Vanoni, Assoc. M. Am. Soc. C. E., and James Daily; and Hunter Rouse, Merit P. White, and W. J. Manetta, Assoc. Members, Am. Soc. C. E.

APPENDIX *A*

The Asymptote, $\mathbf{R}_0$, and the Function of D

The distributional expression for **R** given in Eq. 9*d* represents a range of values from $v = \infty$ and $z_i = 0$ to $v = 0$ and $z_i = \infty$. The value of $\mathbf{R}$ ($= \mathbf{R}_0$) representing the asymptote, however, corresponds to a condition in which v has decreased to zero and beyond to minus infinity. Negative velocity represents a reversed direction of flow, that is, flow entering the street from the inlet. Although it may be possible by analytical means to derive the expression for $\mathbf{R}_0$ in a manner similar to that used in determining $K_0 \mathbf{F}_0$, no such method has suggested itself.

The asymptote $\mathbf{R}_0$ does not occur at the value $\mathbf{R} = 0$, for at that point $K\,\mathbf{F} = \frac{a_c}{2\,h}$, a finite quantity, whereas at an asymptote the quantity must be infinite. This, moreover, is a reason that the three-constant form of hyperbolic equation is required rather than the two-constant form representing the "Second Hyperbolic Law":[6]

$$\frac{\mathbf{R}}{K\,\mathbf{F}} = c_1\,\mathbf{R} + c_2 \qquad (51)$$

The Function of D

In Eq. 36*b* the quantity within parentheses represents a product of functions of **R** and various dimensions. Dividing the coefficient of the term involving **R** by the constant term, $\frac{a_0}{2\,h}$, of the function, and rearranging, then yields (see Eq. 5)

$$\frac{K\,\mathbf{F}}{1 + \frac{a_0 - a_c}{a_0}\left(\frac{\mathbf{R}_0}{\mathbf{R} - \mathbf{R}_0}\right)} = \frac{a_0}{2\,h} = \frac{1}{2}\,\frac{h - d - c_s}{h} = \frac{1}{2}\,(1 - D - C_s) \qquad (52)$$

Hence,

$$\frac{2\,K\,\mathbf{F}}{1 + \frac{a_0 - a_c}{a_0}\left(\frac{\mathbf{R}_0}{\mathbf{R} - \mathbf{R}_0}\right)} + C_s = 1 - D \qquad (53)$$

The locus of values of the left-hand member plotted against values of D thus should yield a straight line. In Eq. 53 $\mathbf{R}_0$ is not constant. Values of the left-hand member for the three experimental cases are calculated in Table 8(*a*). When these values are plotted against values of D, the result is a straight line, confirming the arithmetic in calculations of the value of $\mathbf{R}_0$.

If, on the other hand, it is assumed that $\mathbf{R}_0$ is constant and has the values $-4{,}310$ ($d = 0.0313$, $S_f = 0.02$), and $-3{,}140$ ($d = 0.0000$, $S_f = 0.02$), then

[6] "Intermediate Calculus," by Percy F. Smith and William Raymond Longley, Ginn & Co. (Copyright 1931), p. 203.

values of the left-hand member of Eq. 53, as calculated in Tables 8(*b*) and 8(*c*), produce slightly curved graphs when plotted against D. When plotted against D, $\mathbf{R}_0$ is itself a function of D, and it is practically a straight-line function of D^4 and of d^2.

TABLE 8.—Computation of Values of Function of D

No.	d (1)	h (2)	D (3)	C_s (4)	$2KF$ (5)	$\mathbf{R}$ (6)	$\mathbf{R}_0$ (7)	$\mathbf{R}-\mathbf{R}_0$ (8)	$\frac{\mathbf{R}_0}{\mathbf{R}-\mathbf{R}_0}$ (9)
(a) $\frac{a_0 - a_c}{a_0} = \frac{0.0425}{0.0460} = 0.924$; and $c_s = 0.0228$									
1	0	0.0688	0.000	0.332	0.548	13,070	−3,140	16,210	−0.1937
2	0.0313	0.1001	0.313	0.228	0.378	18,030	−4,310	22,340	−0.1928
					0.385	21,035	−4,510	25,545	−0.1764
3	0.0625	0.1313	0.476	0.1738	0.256	24,150	−9,870	34,020	−0.290

Table 8.—(*continued*)

No.	Col. 9 × 0.924 (10)	1 + Col. 10 (11)	$2KF$ ÷ Col. 11 (12)	Col. 12 + C_s (13)	Col. 6 + 4,310 (14)	− 4,310 ÷ Col. 14 (15)	− 3,982 ÷ Col. 14 (16)	1 − Col. 16 (17)	$2KF$ ÷ Col. 17 (18)	Col. 18 + C_s (19)
(a) (*continued*)					(b) $\mathbf{R}_0 = -4{,}310$; and $\frac{\mathbf{R}_0}{a_0}(a_0 - a_c) = 0.924 \times (-4{,}310) = -3{,}982$					
1	−0.1792	0.821	0.668	1.000	17,380	−0.248	−0.229	0.771	0.711	1.043
2	−0.1783	0.822	0.460	0.688	22,340	−0.1927	−0.1783	0.822	0.460	0.688
	−0.1631	0.837	0.460	0.688						
3	−0.268	0.732	0.350	0.524	28,460	−0.1513	−0.1397	0.860	0.298	0.472

Table 8.—(*continued*)

No.	D^2 (see Col. 3) (20)	D^4 (21)	3,140 + $\mathbf{R}$ (see Col. 6) (22)	− 3,140 ÷ Col. 22 (23)	− 2,901 ÷ Col. 22 (24)	1 − Col. 24 (25)	$2KF$ ÷ Col. 25 (26)	Col. 26 + C_s (27)
(b) (*continued*)			(c) $\mathbf{R}_0 = -3{,}140$; and $\frac{\mathbf{R}_0}{a_0}(a_0 - a_c) = 0.924 \times (-3{,}140) = -2{,}901$					
1	0	0	16,210	−0.194	−0.179	0.821	0.667	0.999
2	0.0981	0.00962	21,170	−0.148	−0.137	0.863	0.438	0.666
3	0.226	0.0512	27,290	−0.115	−0.106	0.894	0.286	0.460

APPENDIX *B*

Notation

The following symbols are defined where they first appear and are assembled here for convenience of reference (see Fig. 1 for designation of dimensions):

$A_0 = \frac{a_0}{h}$, the approach-depth number;

a = depth at a distance, z, from the curb, in feet:
$\bar{a}$ = a weighted or average value of a;
a_0 = depth at curb at the approach section, in feet;
a_c = depth at the crown, or center line of street, in feet;
C = a coefficient:
C_0 = an hyperbolic constant;
$C_s = \frac{c_s}{h}$;
c = a coefficient; height of inlet opening, in feet: c_s = fall due to the normal slope of the model between the approach section and the center of the inlet, in feet;
$D = \frac{d}{h}$, the depth-of-depression number;
d = depth of depression, in feet;
e = subscript denoting excess flow, or overflow; height of overflow sill above normal gutter line produced, in feet;
$\mathbf{F}$ = Froude's number, $\frac{\bar{v}^2}{g\,h}$: $\mathbf{F}_M$ = Froude's number representing actual model flow;
h = head; the characteristic dimension; difference in elevation between the water surface at the approach section and the lip of the inlet at its midlength, in feet: h_P = head for prototype;
i = subscript indicating "inlet" or "integral";
K = the dependent variable, $\frac{\bar{z}\,P}{U}$:
$K_M = \frac{\bar{z}\,P}{U}$ for actual model flow;
$K_\nu = \frac{\bar{z}\,P}{U}$ for model dimensions and velocity, but a calculated value of ν;
l = length of upper end of depression (see Fig. 1);
n = width of overflow notch, in feet; an exponent;
R = hydraulic radius, in feet;
$\mathbf{R}$ = Reynolds' number, $\frac{\bar{v}\,h}{\nu}$:
$\mathbf{R}_M$ = Reynolds' number for actual model flow;
$\mathbf{R}_P$ = Reynolds' number for prototype flow;
$\mathbf{R}_\nu$ = Reynolds' number of prototype magnitude, but model dimension and velocity (ν calculated);
S = longitudinal slope of the model, in feet per foot: S_f = longitudinal slope of the approach flume, in feet per foot;
T = the roughness number, $\frac{t}{h}$;
t = height of roughness protuberances, in feet; subscript denoting "total";
$Y_c = \frac{y_c}{h}$, the crown height number;
y_c = the crown height;

z = distance from curb:

z_i = distance from curb to outer boundary of a filament of flow;

z_t = distance from curb to crown;

z_{iM} = distance, z_i, for model;

z_{tM} = distance, z_t, for model;

f, α, β, θ, etc. = function of;

$\kappa = \dfrac{Q}{Q_t}$.

Dimensional Formulas

G = momentum per second at the approach section of flow within the distance z_i from the curb;	$M\,L\,T^{-2}$
g = acceleration of gravity, in feet per second per second $\left(= \dfrac{\gamma}{\rho}\right)$;	$L\,T^{-2}$
P = hydrostatic pressure acting at the approach section on flow within a distance z_i from the curb, in pounds;	$M\,L\,T^{-2}$
Q = rate of flow, in cubic feet per second at the approach section of flow within the distance z_i from the curb:	$L^3\,T^{-1}$
Q_e = excess flow, or overflow;	
Q_t = "total" flow or flow between the curb and the crown;	
Q_P = flow, Q, in a prototype;	
Q_M = flow, Q, in the model;	
U = moment of momentum per second within a distance z_i from the curb at the approach section;	$M\,L^2\,T^{-2}$
v = the apparent average velocity in a vertical line at a distance z from the curb, in feet per second:	$L\,T^{-1}$
$\bar{v}$ = average velocity, or momentum per second per unit mass per second of flow within the distance z_i from the curb at the approach section $\left(= \dfrac{G}{\rho\,Q}\right)$;	$L\,T^{-1}$
$\bar{v}_M$ = average velocity, $\bar{v}$, for the model;	
$\bar{v}_P$ = average velocity, $\bar{v}$, for the prototype;	
$\bar{z}\,P$ = moment of hydrostatic pressure, P;	$M\,L^2\,T^{-2}$
γ = specific weight, in pounds per cubic foot ($= \rho\,g = 62.4$ lb per cu ft);	$M\,L^{-2}\,T^{-2}$
ν = kinematic coefficient of viscosity, in square feet per second $\left(= \dfrac{\mu}{\rho}\right)$:	$L^2\,T^{-1}$
ν_P = actual kinematic viscosity for a prototype;	
ν_ν = kinematic viscosity representing a Reynolds number of prototype magnitude, but of model velocity and dimension;	
μ = coefficient of viscosity;	$M\,L^{-1}\,T^{-1}$
ρ = mass (slugs) per cubic foot $\left(= \dfrac{\gamma}{g} = 1.938\right)$.	$M\,L^{-3}$

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PAPERS

BASIC ECONOMIC CONSIDERATIONS AFFECTING SINGLE AND MULTIPLE PURPOSE PROJECTS

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Synopsis

A practical analysis of a given condition is followed throughout the paper. The writer has defined an exemplary area and discusses data collected as a result of field investigations. The paper is subdivided into two sections in discussing single and multiple purpose projects.

SECTION I.—ANALYSIS OF DATA, FLOOD-CONTROL PROJECTS

1. Introduction

This subject is so complicated that scarcely any set rules are applicable in all cases. For this reason, an exemplification is made of what may be encountered in the collection of flood damage data, and in the determination of the benefits that may result from the construction of proposed flood protection works. The actual collection of data in the field by experienced investigators cannot be stressed too strongly. It matters not what interpretation may be placed on "direct" or "indirect" damages or benefits; if the basic data are not a true representation of facts, the ultimate answer may be much in error.

2. Direct Benefits

Direct benefits resolve themselves into one question: What benefit in dollars and cents will accrue to the owners and others affected? It does not make any difference whether the owners "cash in" now or wait and receive the benefits when such benefits materialize. They may assume many forms, depending on the adaptability of the particular ownership to benefit. A farm may have partial or total flood protection as a result of protective works; woodlands in either case may be improved from nonagricultural to agricultural lands and, likewise, properties unsuited for industries may be made proper locations for industries. The same condition holds true relative to residential and other uses. The item, "direct net flood losses prevented" on certain agricultural

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by July, 1942.

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properties is a function of benefit, but flood losses on properties cannot be capitalized where there are no flood losses. For example, lands may become industrial properties because of flood protection; or, as a result of flood protection, pasture or ranch lands may be converted to higher agricultural use. In many cases, woodlands and wild grass meadow pastures are benefited by occasional overflows. It may be noted, therefore, that the application of a single rule is not feasible. Different rules and methods must be applied to meet the conditions or, in other words, "every tub must stand on its own bottom." In this connection, every report should be a compendium of the actual field facts based on actual inspection by qualified field men. Guesswork, even on a preliminary examination, should not be accepted. It appears evident that direct benefits should be measured along the following general lines:

(*a*) Take credit for all direct prevented net flood losses (crops, buildings, livestock, or other tangibles), after flood protection, hereafter referred to as Item 2(*a*).

(*b*) Take credit for any additional direct net flood losses prevented that may ensue as a result of clearing or conversion of lands to a higher use after flood protection, hereafter referred to as Item 2(*b*).

Discussing these two benefits, Item 2(*a*) should be susceptible of close estimation by the use of proper field personnel making use of existing factual data; Item 2(*b*) involves an element of speculation or guess. A close study of the economic conditions, in general, in the affected area should be made. The unbiased opinions of reputable businessmen and farmers will be of great assistance. Under Item 2(*b*), abstract considerations are involved, and care should be taken to avoid the consideration of Utopian ideas. Credit should not be taken for benefits from developments that could occur only after flood protection is provided as an item of flood losses prevented. The sum total of remaining benefits in the form of flood losses prevented will be taken care of under Item 2(*b*). Simplification of considerations is important. Field men should not be asked to perform the impossible and draw fine-cut lines, thereby producing a distribution that serves no useful purpose.

3. Calculation of Losses and Benefits

Under Item 2(*a*), all flood losses may be reduced to an annual basis and this, in turn, may be capitalized for the increment of value as a result of flood losses prevented. The rate of capitalization should be variable to reflect risks accurately. For example, an area with only partial flood protection will carry a higher rate than one with complete flood protection. Furthermore, this phase of the situation deals with a farmer's or businessman's dollar and should carry a higher rate than consideration of public funds. Consideration is given now to benefits to individuals. For example, *A* applies to a Land Bank or other agency for a loan. His average annual net income is $500 from his farm or property. If $500 is capitalized at 5%, $10,000 is the value of his farm. Does the Land Bank or loaning agency value this farm or property at

$10,000? In all probability it does not, as it will use from 6% to 15% as a rate for capitalization, making the value range from $3,333 to $8,333, which is more in line for a property with a net income of $500 per year. There is an economic consideration to be given capitalization rates that cannot be ignored; and 10% appears to be normally about the correct rate to apply if abnormal hazards are not involved. This discussion refers to increased value by capitalizing the value of prevented flood losses. In applying capitalization to obtain the increment of increase in value, the entire operation must be analyzed to determine exactly what this prevented loss means to the individual in net gain on an operation. (This is a long discussion within itself and will be omitted here.)

Under Item 2(*b*), "annual flood losses prevented" do not apply, and to apply would be erroneous. The benefit must be taken care of by anticipated enhancement in value. It makes no difference whether the owner sells or holds his property, the problem is to determine the present worth of this increase in value. This will be the "benefit." It represents a "capital sum," and it can be reduced to an annual benefit. The question that may present itself is: What about prevented flood losses on lands converted to higher use as a result of flood protection? For instance, a factory or dwelling is built on property falling under Item 2(*b*), and this property is protected from flood losses as a result of the project. In the first place, the owner purchased or already possessed the land. If he purchased it, he paid the increased benefit price obtained as a result of flood protection, and the seller received a capital benefit. If he happened to be the original owner under the project, he has reaped a benefit in like measure. Pursuing the question further: The factory or house is benefiting by the flood protection. However, the house or factory owner has paid the price he would pay in a flood-free area. Therefore, any increased benefits to this house or factory as a result of prevented flood loss must be in the indirect classification. Otherwise, there will be duplication. It should be borne in mind, also, that enhancement due to change in land use that would have occurred if no protection or additional protection had taken place cannot be considered as a project benefit. Although, for the purposes of exemplification, a case is used in which the trend justifies the consideration of future development, no credit is taken nor should be taken on this development for flood losses prevented or for capital gains. Schedule *B*, the "as is, where is" value which represents the average value of the property without flood protection for the same period as the period of proposed flood protection, is used as the basis for determining the proposed project benefits. Observation has been rather conclusive that, in areas in need of flood protection, developments are usually at a standstill, or going backward. It is believed that a *status quo* condition is likely to hold true in most cases in future years as the population of the United States is expected to reach a stationary level within twenty or twenty-five years. Further, the trend toward the elimination of marginal and submarginal lands from private use will probably continue. Many other valid reasons can be enumerated.

4. Summary

Direct benefits equal flood losses prevented, plus present worth of enhancement brought about by changed use or potential changed use; or, direct benefits = Item 2(*a*) + Item 2(*b*).

5. Example of Application

An exaggerated case is taken to attempt to emphasize various considerations in the foregoing discussion. Taking the case of an area of 640 acres owned by an individual or group of individuals in a proposed flood-control area, the "as is, where is" classification and value of this area are given in Table 1. It is entirely probable that conditions may be such that the present "as is, where is" setup should be used as the basis for determining the proposed project benefits. As previously expressed, one is injecting an element of guess or prophecy in estimating a development trend for the area since he must know what periods in the past will be representative of the future. Fifty years hence is a long time to project a curve of this nature, with the uncertain or vague data as to what has happened in the past. Fluctuations in real estate and commodity prices are beyond accurate estimation. It might be emphasized further that agricultural development during the past eight years has been confined generally to new areas, whereas the trend points toward retrogression in the older areas. However, in the case being exemplified, the development trends exist, and they are reflected in Table 2, which represents the average conditions of the property without flood protection for the same period as the

TABLE 1.—Average Classification (Schedule *A*) of 640 Acres in a Proposed Flood-Control Area

No.	No. of acres	Unit value (dollars per acre)	Value (dollars)
a	100	90.00	9,000
b	100	70.00	7,000
c	100	30.00	3,000
d	100	12.50	1,250
e	140	7.50	1,050
f	100	4.00	400
g	Buildings and improvements		6,300
	640	Total value	28,000

TABLE 2.—Average Cconditions of the Property Without Flood Protection (Schedule *B*)

No.	Description	Acres	Dollars per acre	Total value (dollars)
a	Cleared and in cultivation, subject to slight flood damage	100	100	10,000
b	Cleared and in cultivation, subject to considerable flood damage	120	75	9,000
c	Cleared and in cultivation, subject to heavy flood damage	180	45	8,100
d	Pasture land, largely meadow; cleared but too severely flooded to cultivate	100	15	1,500
e	Woodland; can be cleared (potential) but has severe flood hazard	40	10	400
f	Nonagricultural woodland; too low to drain, or has poor soil; cannot be considered agricultural land if protected; a timber-growing or pasture proposition, or both	100	5	500
g	Building and all-improvement value on entire tract	..	..	6,500
	Total value	..	..	36,000

proposed project life, which is fifty years in this case. This schedule is the appraisal of the average value of the property without flood protection for the same period as the period of proposed flood protection, and it is used as the basis for determining the proposed project benefits. Since complete flood protection is offered in this case, the following conclusions are reached from a study of the area in Schedule B:

Item a.—All of this area, when protected, will receive benefits through direct flood losses prevented.

Item b.—Due to very favorable location, 20 acres of this area, when protected, will have great potentiality for suburban home sites and will convert to higher use. Therefore, 20 acres will receive a capital gain, leaving 100 acres to receive benefits through direct flood losses prevented.

Item c.—Due to location and shipping facilities, 20 acres of this area, when protected, will have great potentiality as industrial property. These 20 acres will receive a capital gain, and the remaining 160 acres will receive benefits through direct flood losses prevented.

Item d.—This area has only a small flood loss and, when protected, it will remain in *status quo* as pasture land, because the owner requires that much acreage in pasture. Due to the nature of the soils and to insufficient drainage facilities, pasturage is its highest and best use. By the elimination of flood hazards, an increase in the number of livestock ranging on this land can be expected. To use both of these benefits would be duplication. The greater benefit (in this case, benefit through flood losses prevented) should be taken.

Item e.—No appreciable flood losses occur on this area; however, when protected, it will receive a capital gain due to flood protection, because the protection will render it more suitable for cultivation and will improve logging conditions in case the owner retains it in woodland. Consequently, the market value will be increased, thereby effecting a capital gain.

Item f.—On this area flood losses are virtually nothing, and due to soils and the lack of drainage facilities, the land is nonpotential or nonagricultural and will remain in this classification after flood protection. However, when protected, the market value will be increases as a timber-growing proposition, due to more favorable logging conditions, and the area will be more valuable as a woods range for livestock. This area, therefore, will receive a small capital gain.

Item g.—These improvements will create benefits either as capital gains or as flood losses prevented; the greater benefit should govern. In this case, the buildings are located principally on high ground. Little use is being made of some of the buildings, and their value to the farm is little more than salvage value. When the area is protected, these improvements will convert the land to maximum use. Consequently, their value to the farm will increase; this will be a capital gain. This example is selected for exemplification, and it is, no doubt, an unusual case. In most instances, Item g would receive a greater benefit from flood losses prevented, putting it under Item $2(a)$, than from capital gain.

From the foregoing conclusions, the appraisal of the average value of the property for the period of protection is given in Table 3. The increase in the value of the area of Items *a*, *b*, *c*, and *d* of Schedule *C* (Table 3) over the value of the same acreage of Schedule *B* (Table 2) is due to flood losses prevented, and the increase in the value of the remaining area of Schedule *C* over the value of the same acreage of Schedule *B* is due to enhancement in value as a result of changed use or potential changed use. These differences are indicated in Table 4.

TABLE 3.—AVERAGE VALUE OF PROPERTY FOR A GIVEN PERIOD OF PROTECTION (SCHEDULE *C*)

No.	Acres	Dollars per acre	Value (dollars)
a	100	110.00	11,000
b	100	100.00	10,000
c	160	65.00	10,400
d	100	17.50	1,750
e	40	12.00	480
f	100	6.00	600
g	20	200.00	4,000
h	20	175.00	3,500
i	Buildings and improvements		7,170
	640	(Total)	48,900

The enhancement in Items *a*, *b*, *c*, and *d* of Table 4 may not always represent annual flood losses capitalized. The average annual flood losses prevented must be determined from data secured in the area under consideration. In this case, the sum is estimated to be $625.00 (see heading "Summary"). By the construction of proposed flood-control works, the area considered would receive an annual benefit of $625 through average annual flood losses prevented and a total capital gain through flood protection of $5,950. Assuming the life of the project to be fifty years, the present worth of this capital gain of $5,950 would be $5,291.27, using 4.5% rate of interest. The annual benefit of $5,291.27, at 4.5%, would be $238.11. The total direct annual benefit, therefore, would be $625.00 plus $238.11, which equals $863.11.

TABLE 4.—DIFFERENCES DUE TO FLOOD LOSSES PREVENTED, AND INCREASE IN VALUE OF REMAINING AREAS

No.	Enhancement by Prevented Flood Losses		Capital Gain	
	Acreage	Dollars	Acreage	Dollars
a	100	1,000	....	....
b	100	2,500	....	....
c	160	3,200	....	....
d	100	250	....	....
e	....	....	40	80
f	....	....	100	100
g	....	....	20	2,500
h	....	....	20	2,600
i	....	....	....	670
....	460	6,950	180	5,950

6. Direct Flood Losses (Other Items)

Other items of flood damage are numerous, such as damage to livestock, seed, feed, and implements, the cost of exodus and reoccupation, damage to business operation through curtailment, inability to fulfil contracts, increased overhead, and damage to chattels of all kinds not heretofore mentioned. In some cases, there may be labor losses but, in such cases, considerable care should be taken not to duplicate this item with indirect losses applying to the area as a whole. The aforementioned losses usually may be cataloged and treated under Item 2(*a*) (flood losses prevented). However, other than farming operations, these losses can best be handled separately and may (and usu-

ally do) consist of both Item 2(*a*) (flood losses prevented) and Item 2(*b*) (outright capital gains as a result of flood protection). Other direct flood losses may consist of physical damage to highways, railroads, and utilities of all kinds and also, when justified, cost borne by federal, state or quasi-public or private agencies in flood fighting and relief work of all kinds, including exodus and reoccupation. Ordinarily, these losses will fall under Item (2*a*) (flood losses prevented).

7. Indirect Flood Losses

No doubt more care should be exercised in the evaluation of indirect flood losses than in dealing with direct losses. Certainly, more experience is required, because the appraiser must have a clear economic picture of the area as a whole. In no sense can justification be found whereby an arbitrary percentage of direct flood losses can be taken as a correct value for indirect losses. Usually, these losses are not inventoried as easily as direct losses where there are factual data available as a result of prior floods. In most cases, the principal items would consist of: (1) Inconvenience caused by disruption of the func-

Fig. 1.—Among Indirect Flood Damages Are the Economic Losses to the Community as a Whole on Account of Business Cessation or Disruption

tioning of railroads, highways, and utilities with the attendant monetary losses; (2) economic losses to the community as a whole on account of business cessation or disruption not specifically included in direct losses; (3) any increased cost of medical care not included in indirect losses; and (4) labor losses as affecting the community as a whole and not cataloged under direct losses. Consideration should be given to the cost of extra labor used in rehabilitation as an offsetting factor. Contiguous and sometimes remote areas may be materially benefited. However, as a result of a project, consideration should be given to the economic effect of taking large areas out of productive use, and

out of taxation. If credit is fully taken for community benefits in indirect benefits, it would appear that proper charge should be made at least for any reimbursement the federal government would make to political units as a result of property taken out of taxation. One can readily see that real field fact-finding must be resorted to in order to form any kind of intelligent estimate of this situation which, for purposes of convenience, is placed under the same heading as "Indirect Losses." The benefit is more suitably cataloged under "Capital Gains."

8. Intangible Losses

Certain flood losses or inconveniences are not susceptible of evaluation in dollars and cents. In this category may be loss of human life and the abnormal spread of disease affecting the health and happiness of inhabitants, disruption of schools and religious assemblies, burial of the dead, and, perhaps, many other inconveniences of more or less importance to a specific community.

9. Intangible Benefits

Benefits accruing to the community, county, state, or nation, as a result of project works according recreational facilities such as hunting, fishing, bathing, or other recreational activities, and wild life facilities, are not susceptible of monetary evaluation. These benefits, however, should be investigated, and probable usage outlined, as a result of intelligent study and investigation.

10. Summary

In this section (I) it has been emphasized that the accurate collection of field data is of paramount importance. No detailed discussion of technique and method is outlined as this is quite an extended discussion within itself. It may be well to mention here that ordinarily a much greater zone or segment of a proposed project would be studied as an entity than is exemplified in the foregoing, depending upon varying conditions of terrain as related to topography, culture, and other features. Belts of several thousand acres are usually inspected in the field and data are compiled, using the most convenient and applicable methods. Technique and method used in the field will be largely governed by maps and other data available and the degree of accuracy required in the findings. It is emphasized that the damage and valuation figures presented in Section I must be based actually on field data obtained by competent personnel.

TABLE 5.—Annual Direct Benefits for Each Classification of 640 Acres Exemplified

No.	Acreage	Average Annual Benefits Through: Losses prevented	Capital gain
a	100	$ 85.00	$ 40.02
b	100	225.00	100.05
c	160	300.00	128.06
d	100	15.00	10.00
e	40	0	3.20
f	100	0	4.00
g	20	45.00	100.05
h	20	37.50	104.05
i	....	20.00	26.81
....	640	$727.50	$516.24

Annual direct benefits of $625 through flood losses prevented were determined from field data. Table 5 gives the annual direct benefits through flood

losses prevented and through capital gain for each classification of the 640 acres exemplified. It shows that the benefits through flood losses prevented for Items *a*, *b*, *c*, and *d* are greater than the benefits through capital gain, whereas the benefits through capital gain for Items *e*, *f*, *g*, *h*, and *i* are greater than the benefit through flood losses prevented. It has been emphasized, throughout the analysis, that the greater benefit should govern. Consequently, the total average annual direct benefits would be as follows:

Average annual direct benefits due to:	
Flood losses prevented = Items *a*, *b*, *c*, and *d*	$625.00
Capital gain = Items *e*, *f*, *g*, *h*, and *i*	238.11
Total average annual direct benefits	$863.11

Conclusion.—On the area of 640 acres exemplified, flood protection would result in a direct annual benefit of $625 and a total capital gain of $5,950. Assuming the life of the project to be 50 years, the present worth of the capital sum enhancement of $5,950, using 4.5% rate of interest, would be $5,291.27. The annual benefit of this sum would be $238.11. The total annual direct benefit, therefore, will be $625.00 plus $238.11 which equals $863.11. Before estimating indirect benefits, all of the area affected by the project must be considered. Consequently, it is not practical to make estimates of the indirect benefits on the 640 acres.

SECTION II.—MULTIPLE PURPOSE PROJECTS

Section I discusses factors affecting flood-control projects. It is possible, of course, to encounter projects wherein flood control may not be of primary importance and the other features—irrigation, power, and navigation—may all be considered in determining economic justification. The degree of flood protection afforded must be considered, of course, when combined with irrigation, power and stream regulation or navigation. For this reason it would complicate the subject, no doubt, to pursue a synthetic analysis of the property taken for exemplification under a flood-control project as outlined in Section I. The same reasoning would apply, however, in determining Schedules *A* and *B*, Section I, as heretofore. In all probability, Schedule *C* would assume a different form, depending upon how the property would be affected by the project purposes.

11. Direct Benefits

The same principles will apply in a multiple purpose project as in a single purpose project—there may be both annual direct benefits and capital gains. It must be borne in mind that duplication should be avoided and that the aggregate of the benefits from whatever sources cannot exceed the intrinsic worth of the increased value of the property due to the construction of the project works.

12. Indirect Benefits

Indirect benefits will be more far-reaching under multiple purpose projects as greater outlying areas will be affected. Plentiful and cheap power, of

course, has far-reaching benefits. Water for irrigation purposes may convert large areas of cheap range lands into areas of intensified farming, bringing in settlers with all the attendant community improvements. Numerous other indirect benefits could also be enumerated.

13. Intangible Losses and Benefits

The same general principles will apply to the consideration of intangible losses and benefits as are discussed in Section I.

14. Acknowledgment

The opinions expressed in this paper are entirely the writer's responsibility and have no relationship to the official directives of any agency of the federal government. It is desired to acknowledge the services of H. E. Cox, Land Appraiser, of the Southwestern Division, U. S. Engineer Department, in checking mathematical calculations and collaborating on economic considerations.

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PAPERS

EARLY CONTRIBUTIONS TO MISSISSIPPI RIVER HYDROLOGY

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Synopsis

Recent research in runoff characteristics of the Mississippi River basin has disclosed certain values among the early records which had apparently been neglected, if indeed they had ever been recognized outside the group responsible for assembling them. These data generally are to be found in some form or other in old reports and records of the War Department, mainly in publications of the Corps of Engineers and the Mississippi River Commission.

The fragmentary hydrologic data of the earliest record periods are assembled in this paper. They are extended by derivation, estimation, or comparison with related data from neighboring stations, and tentatively integrated into a continuous record covering 122 years, ending in 1938. Incorporated within the record are the discharge determinations for the 33 years ending with 1860, as published by the late A. A. Humphreys, Hon. M. Am. Soc. C. E., and Henry L. Abbot, with such adjustments as were found necessary to conform to the hydrographs of calendar years. Likewise, other results, official or unofficial, published or unpublished, were taken into consideration in compiling these basic hydrologic data. A sharp distinction has been made between the official quantities and those derived or estimated. However, such controls and cross references as are available have been used freely, so that the final results, as here submitted, are as nearly beyond challenge as it seems practicable to obtain at this time.

Trends of both precipitation and the resulting runoff depths from year to year, or from decade to decade or other record segment, are disclosed, with fair correlation generally for the 5-yr or longer period means, and to a lesser extent, on an annual or even a monthly basis.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by July, 1942.

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Lower Mississippi River Discharge

Heretofore, the oft-repeated inquiry concerning the actual discharge of the Mississippi River in its lower course has brought unsatisfactory results according to the writer's observation; yet the observed, computed, and estimated discharge quantities have been recorded for 33 of the 43 years, 1818 to 1860, inclusive.[2] The fact that these discharge quantities are expressed in cubic feet per year, and thus involve trillions, should be no barrier against their use. Division by the number of seconds in a calendar year readily reduces the quantities to yearly average discharge in cubic feet per second, as shown in Table 1. The use of hydrographs and tabulations of gage heights assisted in the apportionment of discharge, month by month, to conform substantially with the respective yearly quantities. Moreover, both large-scale hydrographs and daily discharge quantities are available for practically the entire years 1851, 1858, and 1859, representing the most notable flood years of the Mississippi River, within the detailed record period up to that time, at two or more stations on the lower river. Likewise, the same information is available for 1852, and for parts of other years at various stations on the main river and its tributaries, in addition to some 30 years of river-stage records, 1817 to 1860, inclusive, depicted on smaller scale for publication in *Professional Paper No. 13*, and recently generalized and further reduced for inclusion in Fig. 1. This record was compiled from several sources:[3, 4] Reports of the Mississippi River Commission;[5] records of rainfall, river-stage and flood phenomena, compiled

TABLE 1.—Mississippi River Discharge at Natchez, Miss.

Date[a]		Humphreys and Abbot[b]	Thousands of Cubic Feet per Second	
From Nov.:	To Oct.:		Computed average	Calendar year[c]
1818	1819	15,400	489	498
1822[a]	1822	20,500	650	636
1822	1823	27,300	866	864
1823	1824	21,200	670	701
1824	1825	18,200	577	561
1827	1828	26,400	835	847
1828	1829	13,700	434	458
1829	1830	20,700	656	593
1830	1831	17,600	558	547
1833	1834	20,300	644	589
1834	1835	17,200	546	611
1835	1836	21,400	677	673
1836	1837	15,500	492	488
1837	1838	15,300	485	484
1838	1839	11,500	365	359
1839	1840	18,900	598	645
1840	1841	21,400	679	675
1843	1844	29,300	927	947
1844	1845	19,000	603	597
1845	1846	15,300	485	516
1846	1847	21,300	676	670
1848	1849	27,000	856	843
1849	1850	24,000	761	727
1850	1851	20,600	653	668
1851	1852	17,800	563	602
1852	1853	22,000	698	642
1853	1854	17,000	539	559
1854	1855	11,000	349	384
1855	1856	14,800	468	461
1856	1857	15,100	479	500
1857	1858	26,000	825	832
1858	1859	21,000	666	701
1859	1860	15,200	481	481
Totals.........		640,200	20,250	20,359
Means.........		19,400	614	617
Mean, last 10 years........		18,050	572	583

[a] From November to October, inclusive, in each case except the second item, which was January, 1822, to December 1822, inclusive. [b] See Footnote 2, in the text; billions of cubic feet per second. [c] Derived from all available hydrographs and from temperature, rainfall, and discharge records.

[2] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, p. 130.

[3] *Ibid.*, Plates V to XVIII and tabulations.

[4] "Rainfall and Runoff of the Mississippi River Basin," by Harry Larson, thesis presented to the University of Iowa in 1933 in partial fulfilment of the requirements for the Degree of Doctor of Philosophy.

[5] Progress Report of the Mississippi River Commission, 1881, Appendix F, Plate 3.

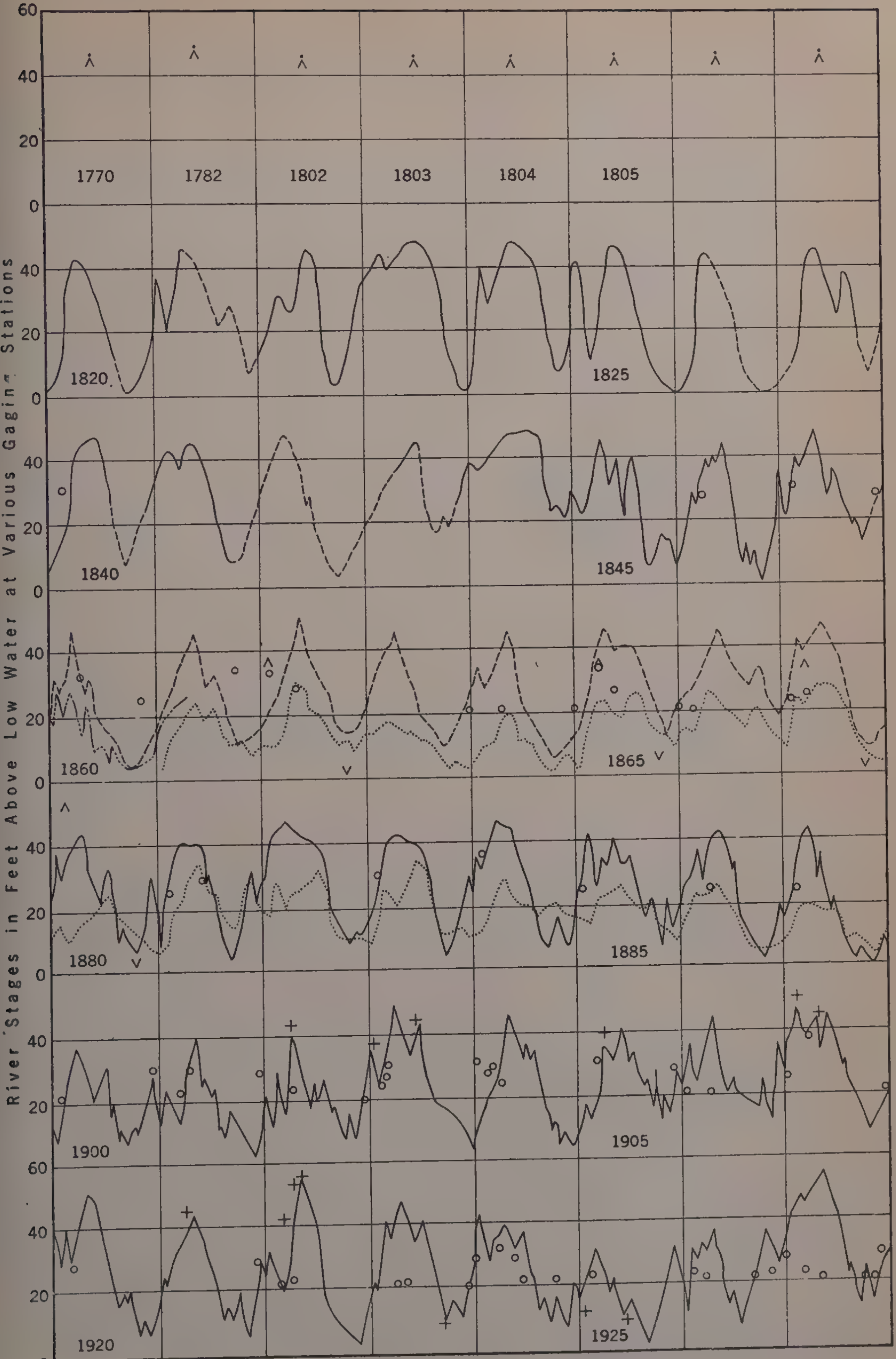
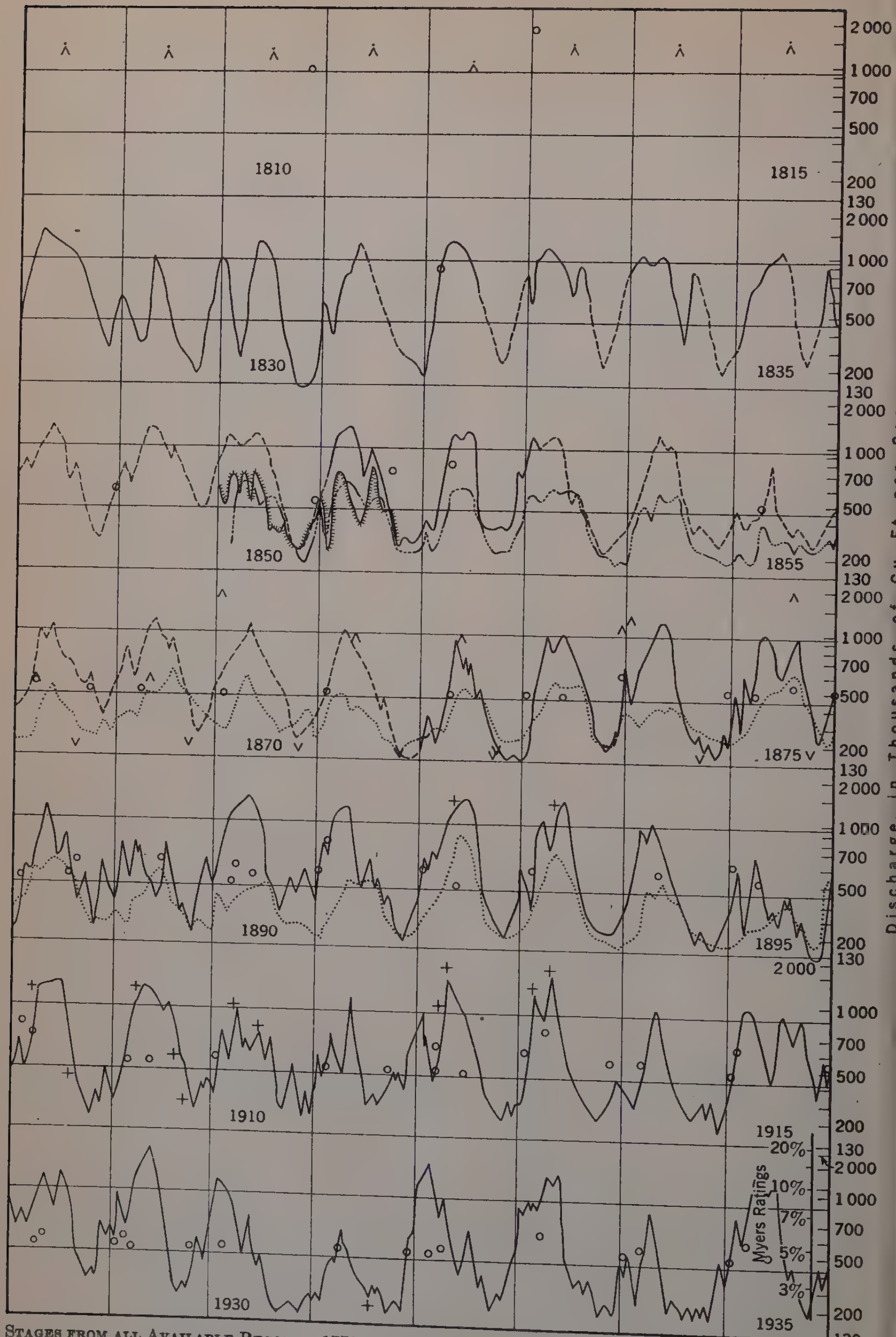


FIG. 1.—MISSISSIPPI RIVER HYDROGRAPHS AND LIMITING



STAGES FROM ALL AVAILABLE RECORDS, 1770 TO 1939

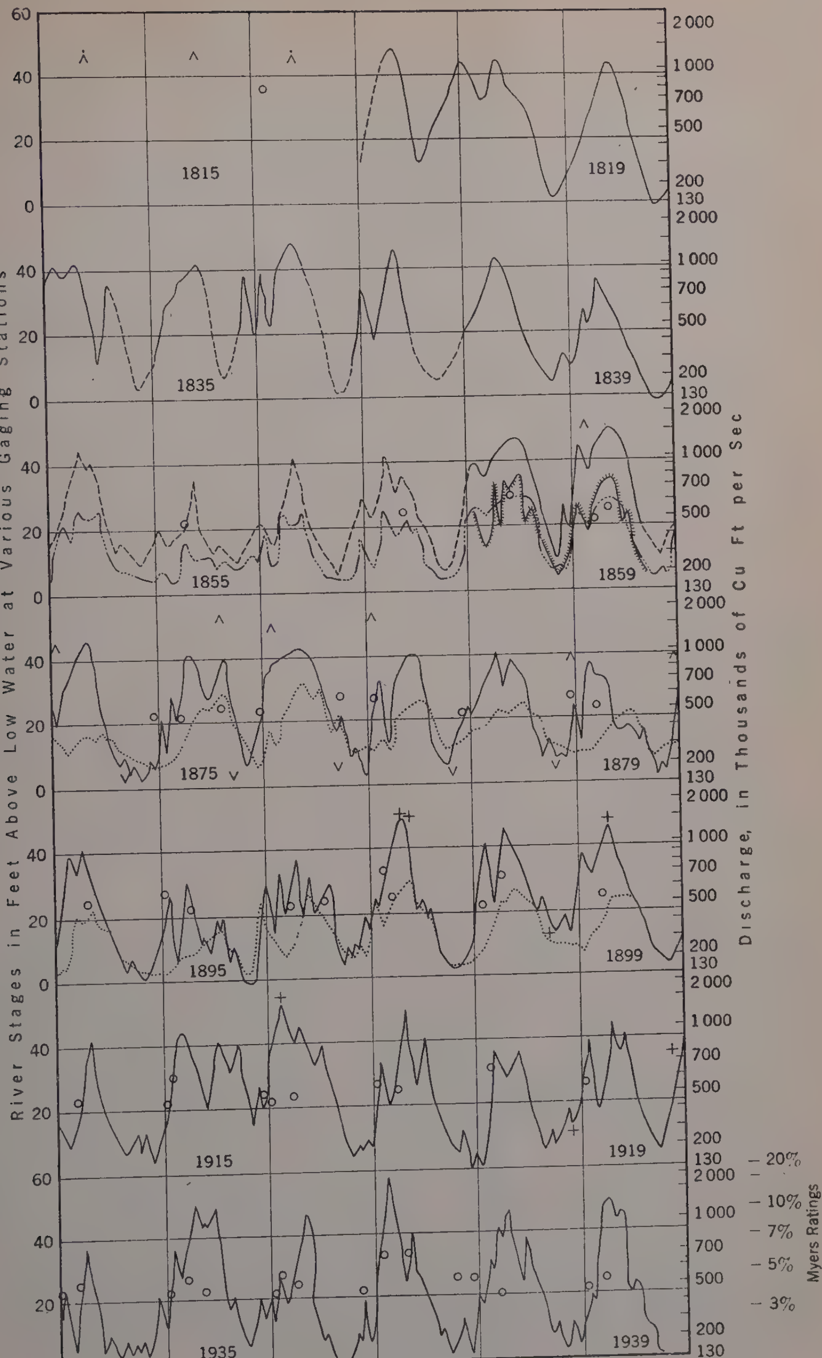


FIG. 1.—*Continued*

by the U. S. Weather Bureau; and the Water-Supply papers of the U. S. Geological Survey.

The key to various lines and symbols in Fig. 1 is as follows:

——— Observed River Stages at Natchez, Miss.
– – – – Derived and Estimated Gage Heights at Natchez, Miss.
········ Monthly Maximum Gage Heights at St. Louis, Mo.
Gage Heights at Memphis, Tenn.
Gage Heights at Donaldsonville, La.
Yearly Maximum Gage Heights at Natchez, Miss.
o Flood-Peaks at Pittsburgh, Pa.
∧ ∨ High and Low River Stages at Cincinnati, Ohio
\+ Divergencies of Maj. Harry Larson's Compilations. from Recent Rating Curves.

The meticulous care with which the observers recorded information such as wind and weather conditions that might possibly affect the operation of the primitive velocity-measuring apparatus (mainly multiple floats and early types of current meter), the faithful portrayal of results showing the influence of the rising as opposed to the falling stage of each major flood event, the recognition of inherent limitations as to attainable accuracy for individual measurements—all are earnest approaches toward, if not the stepping-stones leading to, the more rigorous standards attainable with improved apparatus and methods developed during recent years. Quoting Messrs. Humphreys and Abbot:[6]

> "It is evident that the condition of the river, whether rising or falling, makes a great difference in discharge at any given stand; but it is equally evident that a mean line between these two extremes can be drawn that shall form the basis of a table by which the annual discharge can be deduced from the recorded gauge-readings. For any given day, its indication will be erroneous, but for the entire year, which includes both the rising and the falling branches of the curve, it will be sufficiently accurate * * *."

The difficulties encountered with surface floats, subject to deflection or acceleration due to wind velocity, naturally led to the adoption of multiple floats, largely submerged, and of rods so weighted and adjusted as to maintain them nearly submerged and vertically suspended, supporting small flags above the surface for observation. By such means, Messrs. Humphreys and Abbot developed information regarding depth-velocity relations closely in accordance with the most advanced thought and adopted theories of recent years, as evidenced by the parabolic velocity curves presented by them.[7] Likewise, they presented evidences of a rigorously scientific approach to determinations of both mean and maximum velocities in terms of surface velocities or of those observed some 5 ft below; even a forerunner of the familiar reduction factor of 0.80 or thereabouts, by which observed surface velocity may lead to an approximate value of mean velocity for the station.[8] Abundant proof is to be

[6] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, p. 128.

[7] *Ibid.*, Plate XI.

[8] *Ibid.*, p. 213.

found in the associated text of fair agreement between quantities determined by formula and by observations.

A review of the specific examples in the same report for which detailed calculations are recorded should not fail to impress an unbiased investigator with the idea that those early measurements of stream discharge are beyond challenge, within the reasonable limitations as to accuracy, inherent in their equipment, methods, and working conditions. Whenever hydrologists have had to rely upon quantities determined by slope-area methods, depending on high-water marks observed some days following the passage of a flood peak, whether the event was last month, last year, or 100 years ago, they have occasion to regret the absence of actually observed velocities. Even the progress of uprooted trees in midstream—or better, largely-submerged floats, properly spaced, and observed by theodolites over measured courses in accordance with those early practices—might give results at least more convincing, if not more reliable, than those ordinarily expected by slope-area methods; yet the latter in skilful hands may provide fairly acceptable approximations—often the only available data as to some of the flood-peak discharges officially published during recent years.

Sedimentation and Related Studies

Other basic problems considered in *Professional Paper No. 13* include the development and effects of cutoffs, levee systems, storage reservoirs, spillways, and lateral overflow areas. Also included are the proportion by weight and the distribution of both bed load and sediment in suspension, the latter varying, for the 12-month period beginning February, 1851, from $\frac{1}{681}$ to $\frac{1}{6{,}383}$, or an average of $\frac{1}{1{,}808}$ for the year.[9] Expressed in parts per million by weight, these fractions reduced to 1,467 ppm, 157 ppm, and 553 ppm, respectively, for Carrollton, La., as contrasted with the tentative estimates for the following year—namely, 1,748 ppm, 116 ppm, and 690 ppm. Furthermore, observations extending from March to November, inclusive, 1858, at Columbus, Ky., gave results[10] reducible to 1,493 ppm, 140 ppm, and 757 ppm, whereas those at the Mississippi mouths, taken by George G. Meade (later Major General, U. S. Army) from April to June, 1838, averaged 638 ppm for surface samples, and 785 ppm for subsurface, at depths ranging from 6 ft to 90 ft.

The foregoing observations as to suspended sediment during years of more than normal rainfall and runoff afford interesting comparisons with data published by the U. S. Geological Survey[11] in 1909. These are based on 15 years of sediment determinations by the Corps of Engineers, U. S. Army, and 5 years of determinations by the New Orleans (La.) Water and Sewerage Board, prior to 1907, and one year of determinations at various stations by the U. S. Geological Survey, presumably during years of somewhat less than normal average

[9] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, pp. 147 and 510.

[10] *Ibid.*, p. 139.

[11] *Water-Supply Paper No. 234*, U. S. Geological Survey, Washington, D. C., 1909, p. 87.

runoff, such as prevailed in the period preceding 1907. The report shows (see Table 2)[11] that suspended sediment above and below the Missouri River mouth, at Quincy, Ill., and St. Louis, Mo. (Jefferson Barracks), respectively, was increased some eight-fold, or from 119 to 964 ppm, while the dissolved solids remained nearly constant (see Table 2).

TABLE 2.—SUMMARIZED OBSERVATIONS ON DISSOLVED AND SUSPENDED SOLIDS OF THE MISSISSIPPI RIVER

(Values in Parentheses Were Not Published in Original Tabulation, But Were Derived from Data Contained Therein)

Station	Drainage area, sq miles	Runoff		Solids (ppm)		Tons of Solids per sq mile per yr		Years Required for 1-In. Denudation		
		Cu ft per sec per sq mile	Cu ft per sec	Dissolved	Suspended	Dissolved	Suspended	Dissolved	Suspended	Total
(Above) Minneapolis, Minn.	19,600	0.608	(11,920)	200	7.9	(119)	(4.7)	(1,600)	(40,000)	1,600
Quincy, Ill.	135,500	0.538	(73,000)	203	119	108	63	(1,767)	(3,030)	(1,116)
St. Louis, Mo.	700,700	0.263	(184,300)	206	964	53	250	(3,600)	(764)	(630)
Menard, Ill.	711,900	0.263	(187,000)	269	634	70	164	(2,720)	(1,164)	(816)
Memphis, Tenn.	941,000	0.546	(514,000)	202	519	109	279	(1,750)	(684)	(492)
New Orleans, La.	1,261,000	(0.560)	(706,000)	190	600	(105)	(330)	(1,820)	(580)	(440)
At mouths	1,265,000	0.560	(719,000)	(190)	(600)	105	331	(1,820)	(580)	(440)

DENUDATION—LOCAL AND BASIN-WIDE

Probably the brief chapter on "Denudation," from which Table 2 was obtained,[11] represents the most comprehensive and scientific approach to the problem of which there is evidence, up to the date of its publication. Among the items of interest and value are those purporting to represent the average river discharges and proportions of solids throughout the periods of available record. The fairly consistent relationships between such discharge and sediment computations or estimates, and those developed in connection with this paper and associated research projects, afford at least partial confirmation from officially accepted and published quantities.

One of the most serious errors that might readily result from a casual use of such data would be the assumption that it requires centuries of erosion and runoff to denude the soil a single inch, when it is well known that a day of intense rainfall or of rapid thawing, or a series of storms, may displace recently disturbed, unstable, unprotected soil on moderate slopes to a depth of not only an inch, but occasionally several inches under only normal concentrations of flow. It is well known that soil on sloping shale or other smooth surfaces may slide in its entirety when the plane of cleavage becomes saturated, so that the volume of solids moved locally, for short distances, may far exceed the volume of water releasing it. Thus, the actual depth of local denudation in a semi-arid region, on steep, relatively bare slopes, might represent a large percentage of the actual rainfall and might far exceed the depth of runoff from the entire basin, which usually represents only small percentages of rainfall.

It follows from the foregoing that the solids transported by the Mississippi River and delivered into the Gulf of Mexico are but finely divided samples of the total erosional products—the dissolved or flocculent or other readily suspended portions of the disturbed soil mantle. The heavier and coarser materials, along with varying percentages of the finer elements, are strewn along the flood plains, valley floors, and network of drainage channels, awaiting the next turbulent flow to exact a further toll of easily transported elements.

Runoff from Rainfall and Temperature Data

Without those well-organized approaches to river gagings during the early decades of the nineteenth century, engineers would have been deprived of much helpful hydrologic information as well as scientific observations and opinions in related fields. A fair-minded review of the mass of data yielding the average yearly discharges of the Mississippi River at Natchez, Miss., for record periods of early decades[12] cannot fail to impress one with the magnitude and difficulties of the undertakings, and the foresight, determination, and scientific as well as practical accomplishments of those early observers and authors.

The writer's review of those early records disclosed such consistent relationships between rainfall, temperature, and runoff, all in fair agreement with the same observed phenomena during recent years of record, that they provided access to requisite basic data and indicated a method for deriving probable values of Mississippi River monthly average discharge for the years of rainfall record, including the intermittent and fragmentary observations. One can scarcely use the term "non-record" in this connection, inasmuch as some basic data at important gaging stations are available for every year beginning with 1817. Maximum annual stages at Natchez date back intermittently to 1770,

TABLE 3.—Annual Maximum Gage Readings on the Mississippi River at Carrollton, La.

Year	Maximum gage readings (ft)	Discharge (cu ft per sec)
1811	14.87	1,110,000
12	14.22	1,030,000
13	15.22	1,130,000
14	14.50	1,070,000
15	15.30	1,140,000
16	14.53	1,070,000
17	14.58	1,075,000
18	14.26	1,035,000
19	14.80	1,100,000
1820	14.22	1,030,000
21	14.72	1,095,000
22	14.62	1,085,000
23	15.26	1,135,000
24	15.12	1,120,000
25	14.80	1,100,000
26	14.64	1,090,000
27	14.05	1,017,000
28	15.26	1,135,000
29	13.20	950,000
1830	14.66	1,090,000
31	14.57	1,080,000
32	14.55	1,080,000
33	13.80	1,000,000
34	13.64	990,000
35	14.12	1,020,000
36	15.05	1,120,000
37	14.47	1,070,000
38	14.00	1,015,000
39	12.14	885,000
1840	15.03	1,120,000
41	14.47	1,070,000
42	14.57	1,075,000
43	14.76	1,100,000
44	15.05	1,120,000
45	14.86	1,110,000
46	14.86	1,110,000
47	15.05	1,120,000
48	15.10	1,125,000
49	15.21	1,130,000
1850	13.80	1,000,000
51	15.40	1,150,000
52	14.10	1,020,000
53	15.00	1,115,000
54	14.70	1,095,000
55	9.50	700,000
56	12.80	920,000
57	13.10	945,000
58	15.10	1,120,000
59	15.60	1,170,000
1860	13.40	970,000

[12] *Water-Supply Paper No. 234*, U. S. Geological Survey, Washington, D. C., p. 131.

and continuously after 1801; and available records of maximum gage heights at Carrollton began in 1811, as shown in Table 3.[13]

Table 4 shows some interesting relationships between departures from mean rainfall and runoff, year by year, and month by month, all expressed in inches of depth over the entire drainage area above Natchez. (Runoff was either

TABLE 4.—DEPARTURES OF OBSERVED TEMPERATURE, PRECIPITATION, AND MISSISSIPPI RUNOFF FROM RECORD MEANS

(ΔT, ΔP, and ΔR = Departure of Temperature, Precipitation, and Runoff, Respectively)

Month	1850			1860			Averages, 1817 to 1877		
	ΔT (degrees F)	ΔP (in.)	ΔR (in.)	ΔT (degrees F)	ΔP (in.)	ΔR (in.)	ΔT (degrees F)	ΔP (in.)	ΔR (in.)
January	+6.1	+2.0	+0.36	+2.7	−0.4	+0.06	+0.1	−0.34	+0.01
February	+2.4	+1.0	+0.37	+2.8	+0.4	−0.06	+0.7	+0.17	−0.03
March	−0.3	+0.6	+0.14	+15.7	−2.2	+0.27	+0.3	−0.14	+0.01
April	−6.4	+0.9	+0.09	+2.9	−0.6	−0.35	+0.6	−0.00	−0.01
May	−3.6	−0.8	+0.23	+5.1	−1.4	−0.29	+0.2	−0.04	+0.05
June	+1.8	+1.3	+0.23	+1.2	−0.8	−0.32	+0.8	+0.24	+0.05
July	+2.0	+0.6	+0.09	+3.3	−0.7	−0.25	+0.7	+0.30	+0.04
August	+4.0	+0.5	+0.03	+0.8	−0.7	−0.11	+0.1	+0.13	+0.07
September	−0.5	−1.4	−0.03	−2.8	−0.3	−0.09	−1.2	+0.01	+0.04
October	−2.2	−0.9	−0.03	+0.1	+0.1	−0.05	−2.2	+0.01	+0.02
November	+2.4	+0.3	+0.05	−4.5	+0.9	−0.07	−1.2	+0.18	+0.02
December	−3.1	+1.4	+0.08	−4.4	−0.1	−0.07	−1.1	−0.02	+0.03
Total	+0.2	+5.5	+1.61	+1.9	−5.8	−1.33	−0.2	+0.50	+0.30

measured or estimated for the Mississippi River at Natchez. Temperature observations were made at St. Paul, Minn., from 1819 to 1837, and at St. Louis, from 1838 to 1877. Precipitation was observed at Marietta, Ohio, from 1817 to 1829, and at additional stations thereafter as they became available with desirable distribution. In 1850, the total was 15 stations; in 1860, 22 stations; in 1868, 27 stations; and in 1873, 45 stations.) The two representative years selected for inclusion were 1850 (above normal), and 1860, with subnormal rainfall and runoff. Thus a yearly excess of 5.5 in. in precipitation produced an excess runoff depth of 1.61 in., whereas the deficiency of 5.8 in. produced a departure of − 1.33 in. in runoff. For the first 61 years of the record, the mean rainfall and runoff departures yearly were + 0.50 and + 0.30 in., respectively, from which it may be logically assumed that the same quantities with opposite signs would apply for the latter half of the record, if worked out on a comparable basis. Results actually computed on a yearly basis, but with the addition of several more rainfall station records, were − 0.35 in. and − 0.27 in. for the 61-yr period, 1878 to 1938.

For the month of March, 1860, the influence of the 2.2-in. deficiency in rainfall was more than offset by the unseasonable temperature, the + 15.7° F having released the ice sufficiently to produce an excess depth of 0.27 in. in runoff for that month. Likewise, a rainfall deficiency of 0.8 in. for May, 1850, in combination with a temperature departure of − 3.6° produced a runoff excess of 0.23 in. In many instances the time lag between rainfall and runoff accounts

[13] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, p. 443 and Plate XIV

for the excess or deficiency occurring in the following month or year instead of being in the same period as the rainfall.

Table 5 lists the average rainfall depths for the available stations, to the number of 50, for the entire 122 years of record, together with the departures in both precipitation and runoff, all measured in inches over the drainage basin above Natchez. In general, about 3 in. of rainfall departure produced 1-in. departure of the same sign in runoff. However, there are wide variations from that ratio, and a number of sign reversals.

The Vicksburg-Natchez Record of Mississippi River Discharge

In spite of the 4,900 sq miles excess of drainage area at Natchez as compared with that at Vicksburg, Miss. (representing 0.43% difference), the annual yields should be within 1% of equality. Therefore, they should be almost interchangeable for the early records, if not for the later ones, in view of the admitted uncertainties of stream measurements at their best, requiring a tolerance of some multiple of that difference, according to conditions. Where stream gagings are numerous and of a high standard, it may appear reasonable to take the difference of 1% into account, as has been done for the Vicksburg-Natchez record. Thus, the 122-yr means have reduced to 593,000 cu ft per sec and 599,000 cu ft per sec, respectively. From Table 1, the 33-yr record mean for Natchez was either 614,000 or 617,000 according to the two separate determinations, thus differing by 0.5%, which happens to be the same as the percentage of record-period represented by the two-month difference between those water years and the calendar years. According to Table 6, the first 50-yr mean discharge at Natchez was the maximum attained, or 626,000 cu ft per sec, as compared with 605,000 for the first 100 years, or 602,000 cu ft per sec for the first 110 years. (Table 6 was compiled from the Humphreys-Abbot report; from stages and discharge observations of the Lower Valley of the Mississippi River, January 1 to June 30, 1937; from annual reports of the Mississippi River Commission and the Corps of Engineers; and from Water-Supply papers of the U. S. Geological Survey.)

Consumptive Use of Water

Obviously the increased consumptive use of water in connection with initial fillings of large storage reservoirs, together with the subsequent diversion for irrigation and other purposes, may account for some of the difference between the first 50-yr mean and the mean discharges for later periods at Natchez. It has been computed and estimated that, as of 1930, some 13,000 cu ft per sec were withheld from the Lower Mississippi River for irrigation alone, exclusive of initial storage and attendant losses.[14]

Relative Permanency of Gaging Sections

Opinions expressed by Messrs. Humphreys and Abbot as to the aggradation, degradation, or relative permanence of Mississippi River channel sections were evidently based on more than those early determinations, covering some 44

[14] *Proceedings*, Am. Soc. C. E., October, 1941, p. 1502, Table 14.

years of intermittent observations and records at various stations. Their several years devoted primarily to the investigation, including residence and field work, must have been supplemented prior to the second edition in 1876, because they were still actively engaged in important professional work, and

TABLE 5.—COMPARISONS OF ANNUAL PRECIPITATION
(Units Are

Year of record	Stations	Precipitation[b]: Average total record	Precipitation[b]: Annual observed rainfall	Precipitation[b]: Excess[a]	Runoff excess[a,c]	Year of record	Stations	Precipitation[b]: Average total record	Precipitation[b]: Annual observed rainfall	Precipitation[b]: Excess[a]	Runoff excess[a,c]
1817	1	42.13	(52.00)	9.87	1.42	1852	17	41.59	44.30	2.71	0.11
18	1	42.13	50.92	8.79	0.71	53	16	41.75	36.94	−4.81	0.58
19	1	42.13	36.30	−5.83	−1.13	54	17	41.59	41.00	−0.59	−0.40
20	1	42.13	39.11	−3.02	−1.45	55	16	40.99	41.43	0.44	2.49
1821	1	42.13	43.32	1.19	0.45	1856	21	40.56	35.91	−4.65	−1.57
22	1	42.13	43.38	1.25	0.51	57	22	40.91	39.25	−1.66	−1.11
23	1	42.13	49.34	7.21	3.22	58	24	41.09	49.15	8.06	2.84
24	1	42.13	(53.00)	(10.87)	1.28	59	24	41.09	42.56	1.47	1.28
25	1	42.13	(37.00)	(−5.13)	−0.38	60	22	40.23	34.01	−6.22	−1.33
1826	1	42.13	41.60	−0.53	−1.87	1861	20	38.54	39.76	1.22	0.21
27	1	42.13	41.46	−0.67	−0.49	62	19	37.95	40.00	2.05	0.55
28	1	42.13	49.50	7.37	3.02	63	19	37.95	36.90	−1.05	0.06
29	1	42.13	39.52	−2.61	−1.61	64	21	38.54	32.62	−5.92	−0.56
30	2	41.45	32.35	−9.10	0.00	65	21	38.54	47.64	9.10	1.50
1831	3	41.45	45.50	4.05	−0.55	1866	21	38.54	43.85	5.31	1.44
32	3	41.45	44.56	3.11	1.05	67	27	37.85	37.58	−0.27	1.16
33	3	41.45	37.12	−4.33	0.80	68	27	37.85	40.87	3.02	0.23
34	3	41.45	35.63	−5.83	−0.05	69	27	37.85	40.42	2.57	1.01
35	4	41.08	41.82	0.74	0.21	70	27	37.85	34.57	−3.28	−0.11
1836	6	42.80	42.87	0.07	0.95	1871	27	37.85	35.67	−2.18	−0.96
37	8	36.23	36.88	0.65	−1.25	72[d]	27	37.85	35.08	−2.77	−2.37
38	9	36.53	31.33	−5.20	−1.30	73[d]	45	36.30	38.86	2.56	−0.11
39	10	38.67	33.82	−4.84	−2.78	74[d]	45	36.30	37.64	1.34	−0.14
40	13	42.00	41.55	−0.45	0.62	75[d]	45	36.30	42.42	6.12	0.43
1841	13	42.00	41.60	−0.40	0.98	1876[d]	45	36.30	41.20	4.90	1.51
42	13	42.00	39.30	−2.70	−0.07	77[d]	45	36.30	40.25	3.95	−0.30
43	15	42.44	44.53	2.09	1.04						
44	15	42.44	41.92	−0.52	4.22	Subtotals				45.89	12.91
45	15	42.44	42.20	−0.24	0.05					−33.40	−11.45
1846	14	40.18	44.29	4.11	−0.92						
47	14	40.18	45.29	5.11	0.92	1872	40	35.74	33.31	−2.43	−2.37
48	14	40.18	43.79	3.61	1.77	73	40	35.74	36.77	1.03	−0.11
49	15	41.71	48.51	6.80	2.98	74	40	35.74	36.02	0.26	−0.14
50	15	41.71	47.08	5.37	1.59	75	42	34.95	38.80	3.85	0.43
1851	14	41.90	38.54	−3.36	0.89	1876	42	34.95	38.74	3.79	1.51
Subtotals				82.26	28.68	77	42	34.95	38.30	3.35	−0.30
				−54.76	−13.85						

[a] A minus sign denotes "deficit." [b] Values in parentheses were not published in the original tabulation, but were derived from data contained therein. [c] Average discharge of the Mississippi River at Natchez, Miss., for 122 years (1817 to 1938, inclusive) was about 599,000 cu ft per sec, representing an average yearly runoff depth of 7.06 in. [d] Using the month-by-month analysis from 27 or 45 rainfall stations for comparison with the succeeding six items.

thus were able to participate in the additions, with nearly a 60-yr record before them. Therefore, it seems evident that they had considerable basis for their opinions, repeatedly expressed in their report, to the effect that in general the Mississippi River bed in its lower course was neither building up nor cutting

down, but that the indurated bluish clay through which the channel is carved seems resistant to erosion so long as it remains saturated. They found that the sediment was being swept consistently forward during moderate and high flows, to extend the delta, although local deposition and erosion might occur

ND RUNOFF, MISSISSIPPI RIVER
ches)

ar of cord	Stations	Precipitation[b] Average total record	Annual observed rainfall	Excess[a]	Runoff excess[a,c]
878	42	34.95	36.76	1.81	−0.44
79	42	34.95	34.93	−0.02	−1.74
80	44	33.66	38.11	4.45	−0.46
1881	44	33.66	36.62	2.96	−0.02
82	44	33.66	39.11	5.45	1.63
83	44	33.66	39.62	5.96	0.62
84	43	34.13	39.35	5.22	0.99
85	43	34.13	34.63	0.50	0.20
1886	43	34.13	32.45	−1.68	−0.45
87	42	34.52	31.47	−3.05	−1.56
88	42	34.52	37.37	2.85	−0.25
89	42	34.52	31.29	−3.23	−1.55
90	42	34.52	37.04	2.52	2.06
1891	42	34.52	34.91	0.39	0.04
92	42	34.52	36.02	1.50	1.31
93	50	34.73	32.61	−2.12	−0.01
94	50	34.73	28.22	−6.51	−2.07
95	50	34.73	29.21	−5.52	−3.61
1896	50	34.73	34.63	−0.10	−1.63
97	50	34.73	33.77	−0.96	0.47
98	50	34.73	37.66	2.93	0.35
99	50	34.73	32.17	−2.56	−0.05
1900	50	34.73	34.45	−0.28	−1.69
1901	50	34.73	30.95	−3.78	−2.17
02	50	34.73	36.48	1.45	−1.01
03	50	34.73	33.63	−1.10	1.67
04	50	34.73	29.93	−4.80	−0.81
05	50	34.73	38.20	3.47	−0.55
1906	50	34.73	35.76	1.03	0.07
07	50	34.73	34.20	−0.53	1.75
08	50	34.73	33.66	−1.07	1.48
09	50	34.73	38.25	3.52	0.62
10	50	34.73	29.39	−5.34	−1.41
1911	50	34.73	35.40	0.67	−1.58
Subtotals				46.68 −42.65	13.26 −23.35

Year of record	Stations	Precipitation[b] Average total record	Annual observed rainfall	Excess[a]	Runoff excess[a,c]
1912	50	34.73	34.41	−0.32	0.77
13	50	34.73	34.36	−0.37	0.30
14	50	34.73	31.32	−3.41	−2.57
15	50	34.73	38.36	3.63	0.99
1916	50	34.73	33.07	−1.66	1.55
17	50	34.73	30.77	−3.96	−0.29
18	50	34.73	33.44	−1.29	−2.14
19	50	34.73	37.55	2.82	0.67
20	50	34.73	34.39	−0.34	1.34
1921	50	34.73	35.32	0.59	−0.57
22	50	34.73	33.76	−0.97	0.35
23	50	34.73	36.70	1.97	0.23
24	50	34.73	30.87	−3.86	−0.07
25	50	34.73	30.43	−4.30	−2.57
1926	50	34.73	35.25	0.52	−0.70
27	50	34.73	38.73	4.00	3.72
28	50	34.73	35.30	0.57	1.23
29	50	34.73	36.90	2.17	2.09
30	50	34.73	27.16	−7.57	−1.71
1931[e]	50	34.73	32.34	−2.39	−3.55
31[f,g]	28	37.85	35.95	−1.90	−3.40
32	50	34.73	34.91	0.18	−0.41
33	50	34.73	33.73	−1.00	−0.25
34	50	34.73	29.30	−5.43	−3.21
35	50	34.73	35.89	1.16	1.00
1936	50	34.73	29.30	−5.43	−2.20
37	50	34.73	34.28	−0.45	0.38
38	50	34.73	35.43	0.70	−0.41
Subtotals				18.31 −42.75	14.77 −20.37
1926[g]	28	37.85	39.57	1.72	−0.70
27[g]	28	37.85	45.40	7.55	3.72
36[g]	28	37.85	31.43	−6.42	−2.20
37[g]	28	37.85	38.21	0.36	0.38
122-yr totals				193.14 −173.56	69.62 −69.02
122-yr means				1.58 1.42	0.57 0.57

Last of a 40-yr series evaluated by Maj. Harry Larson (see footnote 5). [f] Incorporating U. S. Geological urvey data on July 1, 1931. [g] Using the month-by-month analysis from 28 rainfall stations for comparison ith corresponding preceding items.

alternately along the channel. This opinion is certainly confirmed by authoritative observations and expert findings of later date.[15] Such data show only a slight tendency toward scouring the river bed, ranging from 0 to 4%

[15] See "Basic Data, Mississippi River." *Annex No. 5*, H. R. Doc. No. 798, 71st Cong., 3d Session, 1931, plates 8, 9, and 10.

TABLE 6.—SUMMARY OF ANNUAL AVERAGE DISCHARGES, MISSISSIPPI RIVER, AT NATCHEZ, MISS.
(Units, 1,000 Cu Ft per Sec; Drainage Area, 1,149,400 Sq Miles; P and R = Precipitation and Runoff, Respectively)

Record Period		Averages During the Following Periods of Years:													Progressive Averages		Five-Year Average Excess[b]				No. of Stations	
																	Inches		%			
From:	To:	5	10	15	20	25	30	35	40	50	60	80	100	120	Years	cfs[a]	P	R	P	R	P	R
1817	1821	599	....	....	....	....	....	....	....	....	....	....	....	....	5	599	+2.2	0.0	+5.2	0.0	1	1
1822	1826	646	622	....	....	....	....	....	....	....	....	....	....	....	10	622	+2.7	+0.56	+6.4	+7.8	1	1
1827	1831	605	....	617	....	....	....	....	....	....	....	....	....	....	15	617	−0.5	+0.07	−1.2	+1.0	3	1
1832	1836	649	627	....	625	....	....	....	....	....	....	....	....	....	20	625	−1.1	+0.60	−2.7	+8.3	6	1
1837	1841	535	....	....	....	607	....	....	....	....	....	....	....	....	25	607	−2.2	−0.77	−5.3	−10.7	14	1
1842	1846	672	604	619	....	....	618	....	....	....	....	....	....	....	30	618	−0.3	+0.88	−0.7	+12.2	15	2
1847	1851	737	....	....	....	....	....	635	....	....	....	....	....	....	35	635	+3.2	+1.66	+7.7	+23.1	15	2
1852	1856	535	636	....	620	....	....	....	622	....	....	....	....	....	40	622	−2.1	−0.77	−5.0	−10.7	21	2
1857	1861	631	....	634	....	....	....	....	....	....	....	....	....	....	45	623	+0.4	+0.38	+1.0	+5.3	24	5
1862	1866	649	640	....	....	645	....	....	....	626	....	....	....	....	50	626	+1.2	+0.60	+3.1	+8.3	21	4
1867	1871	622	....	....	....	....	....	....	....	....	....	....	....	....	55	626	−0.2	+0.28	−0.5	+3.9	27	9
1872	1876	587	605	619	622	....	627	....	....	....	622	....	....	....	60	622	+1.2	−0.14	+3.3	−1.9	45	11
1877	1881	540	....	....	....	....	....	....	....	....	....	....	....	....	65	616	+2.5	−0.71	+7.1	−9.9	45	17
1882	1886	649	595	....	....	....	....	602	....	....	....	....	....	....	70	618	+3.1	+0.60	+9.1	+8.3	44	21
1887	1891	581	....	590	....	596	....	....	....	....	....	....	....	....	75	616	−0.1	−0.22	−0.3	−3.1	42	24
1892	1896	497	539	....	567	....	....	....	595	....	....	609	....	....	80	609	−2.6	−1.24	−7.5	−17.3	50	25
1897	1901	546	....	....	....	....	....	....	....	....	....	....	....	....	85	605	−0.9	−0.64	−2.6	−8.9	50	25
1901	1906	588	567	544	....	....	567	....	....	....	....	....	....	....	90	604	0.0	−0.13	0.0	−1.8	50	26
1907	1911	613	....	....	....	....	....	....	....	....	....	....	....	....	95	604	−0.6	+0.17	−1.7	+2.4	50	26
1912	1916	616	615	....	591	572	....	....	....	584	....	....	599	....	100	605	−0.4	+0.20	−1.2	+2.8	50	26
1917	1921	582	....	604	....	....	....	575	....	....	....	....	....	....	105	604	−0.4	−0.20	−1.2	−2.8	50	26
1922	1926	551	567	....	....	....	....	....	....	....	....	....	....	....	110	602	−1.3	−0.58	−3.7	−8.1	50	26
1927	1931	631	....	....	....	....	....	....	....	....	....	....	....	....	115	603	−0.6	+0.38	−1.7	+5.3	50	28
1932	1936	514	572	565	569	....	585	....	580	....	576	....	....	599	120	599	−2.1	−1.03	−6.0	−14.3	50	29
1937	1938	599	....	....	....	....	....	....	....	....	....	....	....	....	122	599	0.0	0.0	0.0	0.0	50	30
1817	1938	599	599	599	599	599	599	599	599	599	599	599	599	599	122	599	....	....	....	....	....	....

[a] Cfs = cubic feet per second. [b] Minus sign denotes "deficiency."

for the 20-yr period, 1894 to 1913, and adhering quite closely to the cross-sectional areas and dimensions shown in *Professional Paper No. 13.*[16] Furthermore, all available official data, whether published or unpublished, were summarized and plotted for successive periods in Figs. 2 and 3, for six important gaging stations on the lower river. These show not only the meanderings of

TABLE 7.—MAXIMUM GAGE HEIGHTS AND DISCHARGES AT MEMPHIS, TENN., AND VICKSBURG, MISS.

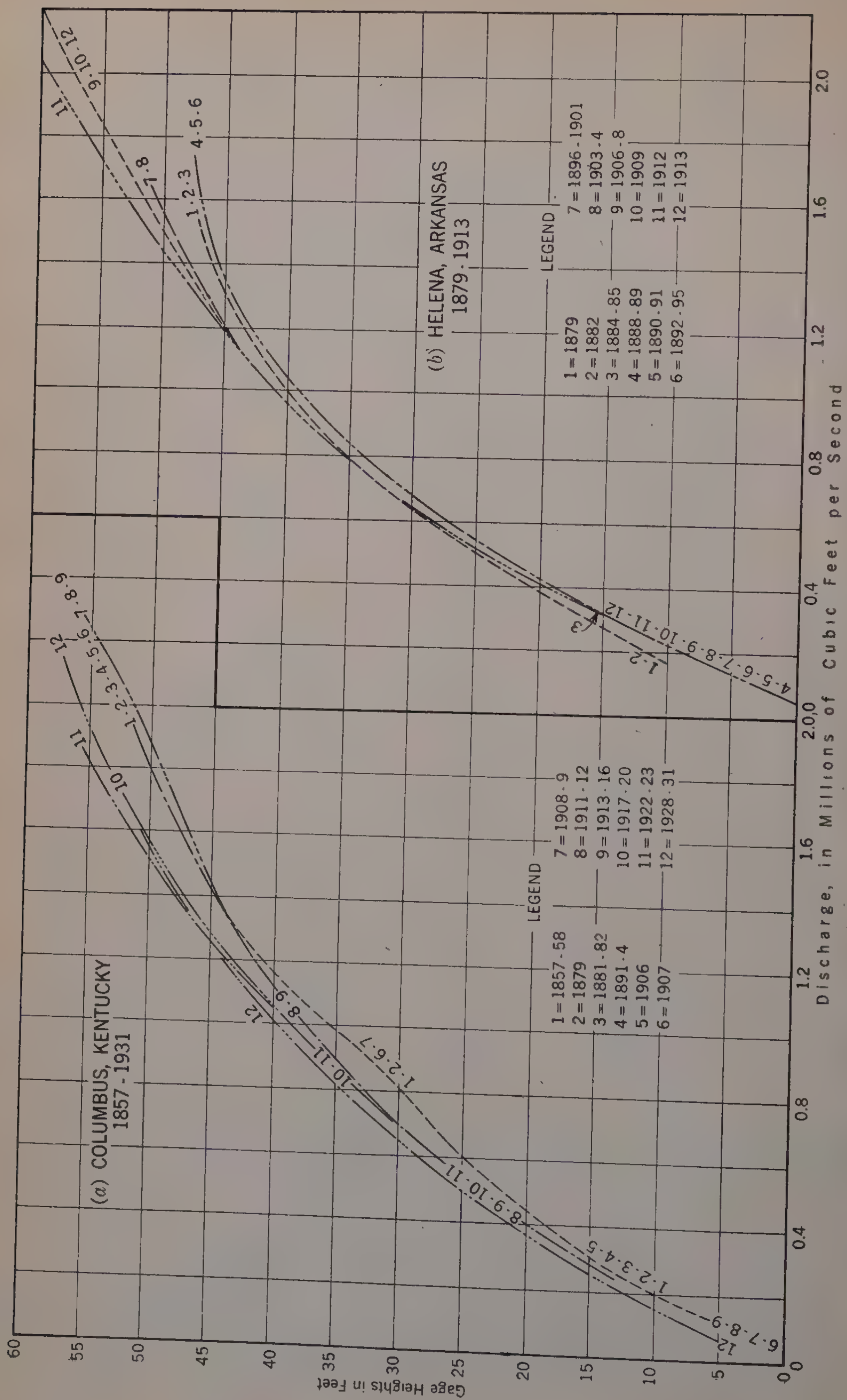
No.	Date	Gage readings (ft)	THOUSAND CUBIC FEET PER SECOND[a]		
			Discharge	Storage[b]	Maximum[c]
1	June 23, 1858[d]	35.3	(1,300)	(180)	(1,480)
2	June 26, 1858	46.98[a]			(1,400[a])
3	April 27, 1862	51.10[a]			(1,700[a])
4	March 20–21, 1882	48.75[a]			(1,540[a])
5	March 25, 1884	49.00[a]			(1,550[a])
6	March 16–17, 1890	35.60	1,400	170	1,570
7	April 24–25, 1890	49.05[a]			(1,550[a])
8	April 2–4, 1891	48.1[a]			(1,490[a])
9	May 10, 1892	48.4[a]			(1,500[a])
10	May 22–23, 1893	48.3[a]			(1,495[a])
11	March 20–21, 1897	37.66			(1,600)
12	April 16, 1897	52.48[a]			(1,800[a])
13	April 10–11, 1898	37.22			(1,600)
14	April 24–25, 1898	49.4[a]			(1,580[a])
15	March 20, 1903	40.10			(1,630)
16	March 28, 1903	51.80[a]			(1,800[a])
17	April 11, 1904	39.20	1,620	....	1,620
18	February 3, 1907	40.30			(1,700)
19	February 12–13, 1907	49.65[a]			(1,700[a])
20	March 22, 1909	38.60			(1,600)
21	April 1–2, 1909	48.00[a]			(1,550[a])
22	April 6, 1912	45.23	(1,800)	240	2,040
23	April 12, 1912	51.65[a]	(1,850[a])	(300[a])	(2,150[a])
24	April 9, 1913	46.55	(2,030)	220	2,250
25	April 27–28, 1913	52.20[a]	(1,900[a])	(300[a])	(2,200[a])
26	February 9, 1916	43.4	(1,750)	....	1,750
27	February 15, 1916	53.85[a]			(1,700[a])
28	April 23, 1917	49.98[a]			(1,730[a])
29	April 27, 1920	50.90[a]			(1,825[a])
30	April 1–2, 1922	42.5	(1,560)	....	1,560
31	April 28, 1922	54.85[a]			1,826[a]
32	April 23–25, 1927	45.8	(1,860)	500	2,360
33	May 4, 1927	58.40[a]	(2,145[a])	(350[a])	2,495[a]
34	July 12–16, 1928	49.30[a]			(1,670[a])
35	May 25, 1929	41.5	(1,600)	....	1,600
36	June 6–7, 1929	55.2[a]	1,741[a]		1,741[a]
37	February 19, 1932	38.7	1,308	....	1,308
38	February 28–29, 1932	50.27[a]	(1,410[a])		1,410[a]
39	April 9, 1933	40.1	1,416	....	....
40	June 10, 1933	47.50[a]	1,360[a]		1,360[a]
41	March 19, 1934	30.0	839	....	....
42	April 13–14, 1934	34.58[a]	877[a]		877[a]
43	April 17, 1936	40.3	1,360	....	1,360
44	April 30, 1936	42.54[a]	1,280[a]		1,280[a]
45	February 7, 1937	50.4	2,020	500	2,520
46	February 17–22, 1937	53.2[a]	2,080[a]		2,080[a]

[a] Discharges marked thus: [a]—are Vicksburg readings, and the remainder are Memphis readings. Values in parentheses are unofficial or estimated quantities. [b] Additional floodway and valley storage discharge. [c] Maximum river and floodway discharge. [d] Item 1 represents the record-period maximum for 40 years or more preceding 1858.

the rating curves, but also the near approach to constancy for considerable periods, and particularly in the lower segments of such curves.

Further evidence concerning the progressive minor changes in rating curves, showing the relation of gage reading to river discharge, is offered in Table 7.

[16] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, Appendix C and Plate X.


(a) COLUMBUS, KENTUCKY
1857-1931
LEGEND
1 = 1857-58
2 = 1879
3 = 1881-82
4 = 1891-4
5 = 1906
6 = 1907
7 = 1908-9
8 = 1911-12
9 = 1913-16
10 = 1917-20
11 = 1922-23
12 = 1928-31
(b) HELENA, ARKANSAS
1879-1913
LEGEND
1 = 1879
2 = 1882
3 = 1884-85
4 = 1888-89
5 = 1890-91
6 = 1892-95
7 = 1896-1901
8 = 1903-4
9 = 1906-8
10 = 1909
11 = 1912
12 = 1913
Gage Heights in Feet
Discharge, in Millions of Cubic Feet per Second

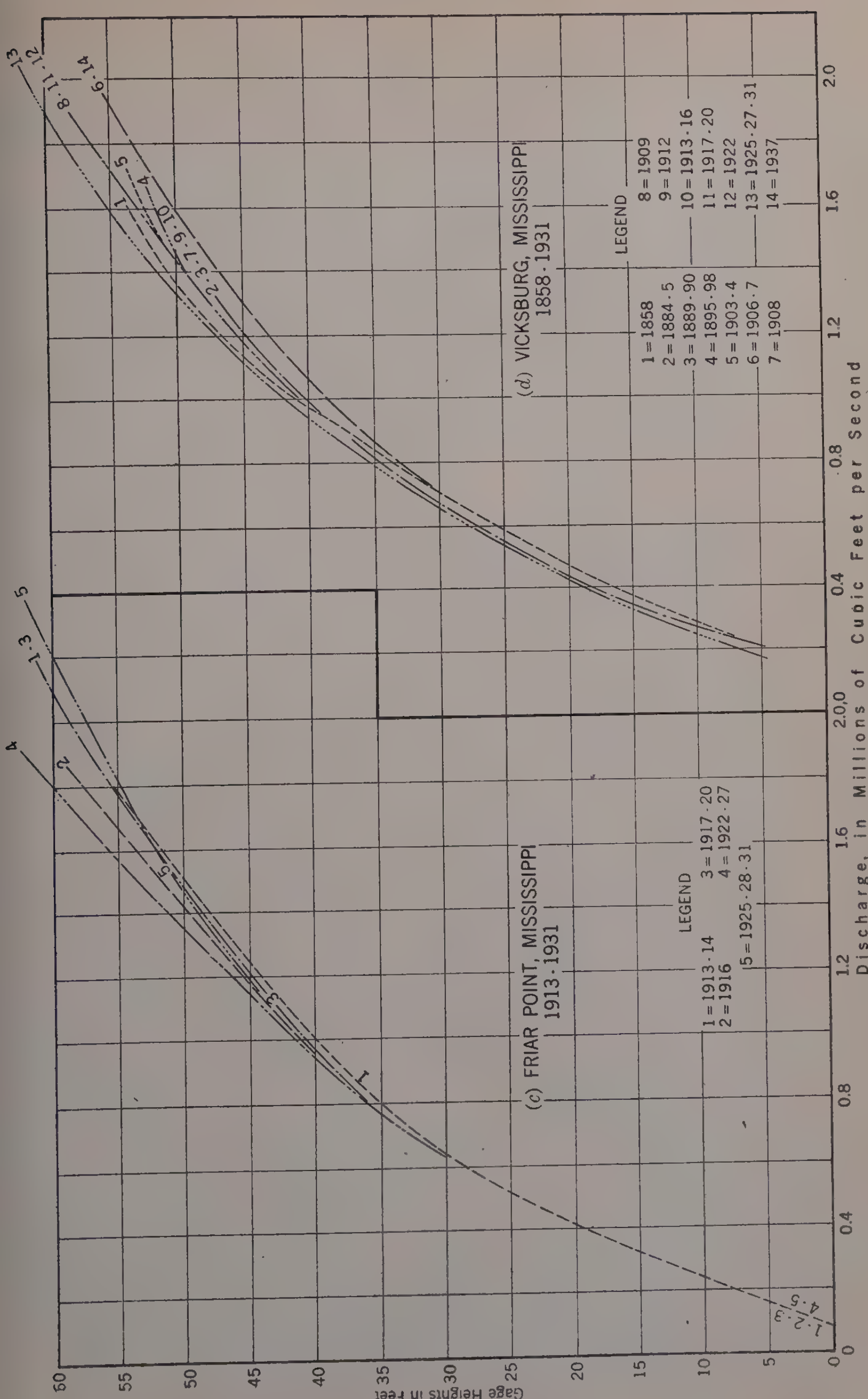


FIG. 2.—DISCHARGE OF THE MISSISSIPPI RIVER COMPILED FROM THE MISSISSIPPI RIVER COMMISSION AND THE U. S. CORPS OF ENGINEERS

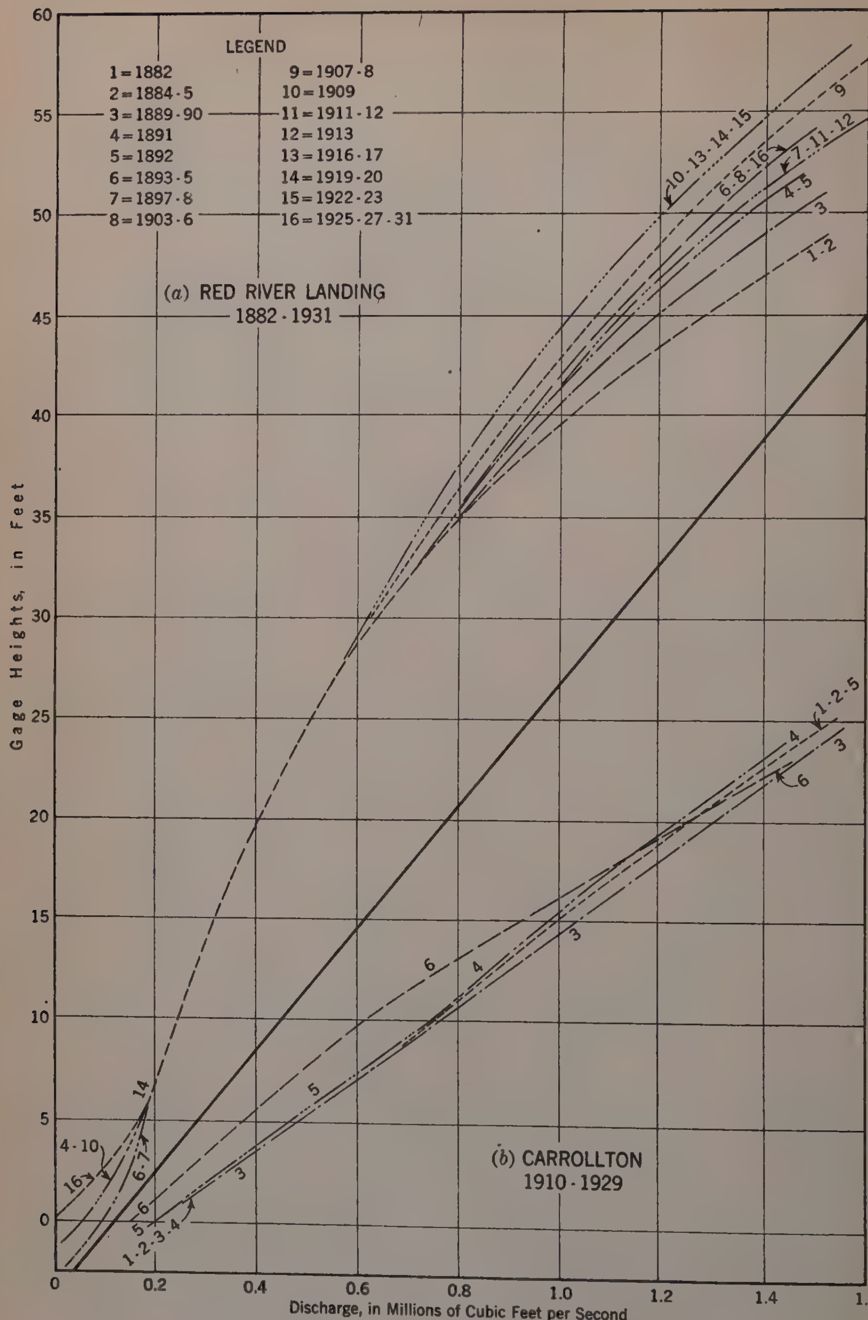


FIG. 3.—DISCHARGE OF THE MISSISSIPPI RIVER, IN LOUISIANA

Table 7 lists the official, derived, and estimated quantities for the principal flood events to date, beginning with the 40-yr (or longer) maxima recorded at both Memphis, Tenn., and Vicksburg. This tabulation indicates that an addition of about 67% to the peak discharge at these two stations produced corresponding increases in maximum gage readings of 15.1 ft and 11.4 ft, respectively. Some undetermined part of these changes must be ascribed to reduced lateral flood-plain storage and total flood-channel section, due to confinement by levees. For Vicksburg, the rise of flood surface due to restrictions by levees was approximately 5 ft, according to data in Fig. 2(*d*). There is shown likewise some evidence of increased channel capacity during early periods, associated with removal of snags or other obstructions to navigation as well as to stream flow; and finally, in the 6-yr period, 1931 to 1937, the influence of cutoffs apparently restored the maximum discharge capacity, as proved by the low position of the rating curve No. 14. This is likewise in accordance with the findings of Brig.-Gen. Harley B. Ferguson,[17] M. Am. Soc. C. E., during his tour as President of the Mississippi River Commission.

Similarly, the writer found:[18]

> "For example, the gage-heights of 46.2 feet corresponding to a discharge of 1,520,000 cfs (Stages and discharge-observations, Lower Valley of the Mississippi River; Mississippi River Commission report, 1937) on March 30, 1891, was exceeded by 4-feet or more for equal discharges during the past 15 years or so."

In Fig. 4 are depicted the yearly, 5-yr, and 10-yr trends of precipitation at Marietta, and the Mississippi River runoff depths in inches over the watershed at Vicksburg, throughout the 123 years of record, including 1939. Apparently a fair correlation is obtained for 92 years, or 75% of the time on a yearly basis, slightly better on the 5-yr basis, and still more uniformly on the 10-yr basis. Likewise, the graphs of cumulative rainfall and runoff departures have several points in common, or nearly so, on the scales adopted; and either the parallelism of trends or their close association and interweaving of lines proves beyond reasonable challenge that this one rainfall station is more of an index to Mississippi River discharge at Vicksburg than would ordinarily be expected. No doubt a part of the explanation lies in the fact that Marietta is near the center of the most productive tributary basin, contributing nearly 50% of the discharge at Vicksburg. The addition of other outstanding station-records as they became available, up to a total of 10, was found to improve the correlation in some parts of the record; but the addition of widely distributed stations to a total of 50 seemed to disturb such correlation of the latter part of the record, as if too much weight was being accorded the areas of low runoff, such as the Great Plains.

For plotting the rainfall and runoff data to afford the most convenient comparison, it was found advisable to utilize a scale for precipitation depths either four or five times the scale adopted for runoff depths, as shown in Fig. 4. For the lower part of the drawing, dealing with cumulative departures, the datum or zero lines do not coincide. In the upper part of the same figure, the mean annual precipitation and runoff depths are slightly separated.

[17] "Effects of Mississippi River Cut-Offs," by Harley B. Ferguson, *Civil Engineering*, December, 1938, p. 828, Fig. 24.

[18] *Transactions*, Am. Geophysical Union, 1939, p. 165.

TABLE 8.—SUMMARIZED DISCHARGE RECORDS AND DERIVED OR ESTIMATI

(Parentheses Designate Unoffic

No.	River	Station	Years of Record: Gag-ings	Years of Record: Gage Hts.	Drain-age area	Mean Discharge for Designat… Water Years, October 1 to Septem… 1933–34	1934–35	1935–36	1936–
1	Column No.		1	2	3	4	5	6	7
2	Units[a]		(yr)	(yr)	(Sq mile)	cfs	cfs	cfs	cfs
3	Allegheny	Franklin, Pa.	20	54[c]	5,982	6,846	8,148	9,451	13,17
4	Kiskiminetas	Avonmore, Pa.	31	....	1,723	2,260	3,162	3,404	(4,37
5	Tygart	Fetterman, W. Va.	31	....	1,304	1.887	2,903	2,891	2,77
6	Cheat	Pisgah, W. Va.	25	25[c]	1,360	2,219	3,383	3,159	3,28
7	Youghiogheny	Connellsville, Pa.	29	66[c]	1,326	1,757	2,686	2,688	3,05
8	Monongahela	Charleroi, Pa.	5	53	5,213	6,260	9,494	9,497	10,32
9	Ohio	Sewickley, Pa.	5	67[c]	19,500	20,960	29,840	32,030	39,52
10	Ohio	Wheeling, W. Va.	....	38	24,800	(23,300)	(34,460)	(37,065)	(47,42
11	Ohio	Parkersburg, W. Va.	....	51	37,914	(29,820)	(48,020)	(51,265)	(68,66
12	Ohio	Huntington, W. Va.	4	25	55,200	(46,030)	82,920	79,370	95,11
13	Ohio	Cincinnati, Ohio	....	68	76,580	(59,000)	(111,900)	(100,600)	(126,00
14	Ohio	Louisville, Ky.	4	67	91,200	(68,200)	(131,600)	111,900	149,00
15	Ohio	Evansville, Ind.	....	46	107,000	(84,000)	(167,100)	(127,900)	(180,80
16	Ohio	Metropolis, Ill.	{12 / 39[c]	80[c]	203,000	(161,000)	(285,200)	226,600	337,10
17	Muskingum	McConnelsville, Ohio	17	26	7,411	2,879	6,607	7,182	11,73
18	Kanawha	Kanawha, W. Va.	61	61	8,367	7,824	16,180	13,840	11,48
19	Scioto	Chillicothe, Ohio	17	31	3,847	883	2,038	2,803	5,16
20	Miami	Hamilton, Ohio	27	29[c]	3,639	1,179	1,397	2,257	5,46
21	Licking	Catawba, Ky.	10	52[c]	3,320	2,240	6,896	3,889	5,39
22	Kentucky	Lockport, Ky.	11	53[c]	6,170	5,346	12,950	8,237	8,89
23	Green	Livermore, Ky.	8	40	7,500	7,791	17,440	7,806	15,06
24	Wabash	Mt. Carmel, Ill.	11	54	28,600	8,043	22,710	14,510	39,25
25	Cumberland	Nashville, Tenn.	50	65	12,860	(16,700)	(23,500)	(16,300)	(24,60
26	Holston	near Rogersville, Tenn.	36	37	3,035	2,848	5,114	4,523	4,01
27	Little Tennessee	McGhee, Tenn.	33	35	2,443	3,909	5,051	6,080	5,71
28	Clinch	below Norris Dam	11	55[c]	2,913	3,200	5,009	4,264	4,63
29	Hiwassee	Charleston, Tenn.	19	56	2,298	3,217	3,732	5,126	4,36
30	Tennessee	Chattanooga, Tenn.	64	65	21,400	(26,000)	35,700	38,600	36,81
31	Tennessee	Florence, Ala.	44	67	30,810	37,400	49,650	55,740	53,57
32	Tennessee	Johnsonville, Tenn.	49	62	38,520	46,140	62,540	61,740	67,70
33	Mississippi	Elk River, Minn.	23	....	14,500	1,454	2,429	2,889	2,96
34	Mississippi	St. Paul, Minn.	48	61	36,800	1,935	3,688	5,330	5,37
35	Mississippi	La Crosse, Wis.	9	64	62,800	11,300	22,930	23,570	17,58
36	Mississippi	Le Claire, Iowa	65	65	88,600	18,870	41,820	37,390	33,16
37	Mississippi	Keokuk, Iowa	60	71	119,000	21,540	57,260	47,680	49,90
38	Mississippi	Grafton, Ill.	39	59	170,000	(36,800)	(106,000)	(72,200)	(86,00
39	Mississippi	St. Louis, Mo.	10	81	701,000	67,700	187,600	113,700	146,80
40	Mississippi	Memphis, Tenn.	9	67	932,800	244,285	511,400	349,900	516,70
41	Mississippi	Arkansas City, Ark.	39	59	1,130,700	286,900	651,500	387,900	623,60
42	Mississippi	Natchez, Miss.	35	105	1,149,400	(308,150)	(683,660)	(405,740)	(647,80
43	Mississippi[b]	..........[b]	45	65	1,242,700	(342,665)	(772,870)	(424,730)	(701,08
44	Minnesota	Mankato, Minn.	35	....	14,600	136	590	1,084	1,60
45	St. Croix	St. Croix Falls, Wis.	33	....	5,930	1,754	3,356	3,803	3,13
46	Chippewa	Chippewa Falls, Wis.	28	....	5,600	2,563	5,993	5,117	3,44
47	Wisconsin	Muscoda, Wis.	24	66[c]	10,300	4,454	11,040	7,419	6,94
48	Rock	Como, Ill.	24	....	8,700	1,741	4,783	3,190	5,93
49	Iowa	Wapello, Iowa	23	....	12,480	1,210	5,622	4,961	6,65
50	Des Moines	Keosauqua, Iowa	25	32	13,900	607	5,288	3,823	4,21
51	Illinois	Peoria, Ill.	28	67	13,480	10,810	18,590	13,320	16,12
52	Missouri	Fort Benton, Mont.	57	57	24,600	4,952	4,307	4,559	3,61
53	Missouri	Hermann, Mo.	39	65	528,200	29,750	80,010	41,090	59,00
54	Osage	Bagnell, Mo.	13	....	14,000	2,381	14,790	3,950	11,63
55	White	Clarendon, Ark.	11	54	25,750	17,709	43,252	10,246	(35,16
56	Arkansas	Little Rock, Ark.	39	67	157,900	18,740	65,400	14,010	35,70
57	Yazoo	Greenwood, Miss.	11	54[c]	7,450	5,720	10,230	(7,000)	(14,90
58	Red	Alexandria, La.	39	67	65,850	(20,085)	42,447	10,885	(27,00

[a] Cfs = cubic feet per second; csm = cubic feet per second per square mile. [b] Mississippi-Atchafalaya, n

UANTITIES AT SELECTED STATIONS IN THE MISSISSIPPI RIVER BASIN
erived, or Estimated Quantities)

ERIODS ENDING SEPTEMBER 30, 1938, AND PERCENTAGES OF LATEST FIVE-YEAR MEANS

	Years of Record; Observed, (Derived), or (Estimated)									
937–38	5		10		15		20		25	30
8 cfs	9 cfs	10 csm	11 cfs	12 %	13 cfs	14 %	15 cfs	16 %	17 %	18 %
10,970	9,717	1.62	9,500	97.8	10,036	103.3	9,747	100.3	(103.9)	(106.0)
(2,765)	3,170	1.84	2,789	88.0	3,003	94.7	2,952	93.1	93.0	95.8
2,833	2,658	2.04	2,447	92.1	2,633	99.1	2,594	97.6	97.1	98.2
3,360	3,080	2.26	2,852	92.6	(3,000)	97.4	(2,990)	97.1	(98.5)	(100.6)
2,410	2,520	1.90	2,311	91.7	2,450	97.2	(2,441)	96.9	96.9	99.2
9,341	8,982	1.72	(8,260)	92.0	(8,870)	98.8	(8,860)	98.6	(98.6)	(101.3)
33,120	31,090	1.595	(30,000)	96.5	(31,600)	101.5	(31,500)	101.3	(101.6)	(101.3)
(39,270)	(36,530)	1.473	(35,130)	96.2	(37,120)	101.6	(36,900)	101.0	(101.6)	(101.0)
(55,490)	(50,880)	1.342	(48,370)	95.1	(51,000)	100.2	(50,600)	99.5	(100.2)	(99.3)
85,240	77,730	1.408	(71,680)	92.2	(73,000)	93.9	(74,000)	95.2	(96.8)	(96.8)
108,700	(100,860)	1.317	(94,700)	93.9	(96,000)	95.2	(98,000)	97.2	(99.0)	(98.8)
124,000	116,920	1.282	(111,220)	95.1	(114,000)	97.5	(116,000)	99.2	(100.5)	(99.2)
144,200)	(140,800)	1.316	(134,000)	95.2	(140,000)	99.4	(143,000)	101.6	(101.3)	(99.4)
269,800	255,940	1.261	253,970	99.2	263,000	102.7	272,000	106.3	104.7	107.4
8,504	7,380	0.996	6,891	93.4	7,200	97.6	(7,250)	98.2	(98.9)	(98.9)
13,170	12,499	1.494	11,561	92.5	11,656	93.3	11,821	94.6	95.3	95.1
3,503	2,879	0.748	3,257	113.1	3,427	119.0	(3,300)	114.6	(110.0)	(109.8)
4,439	2,948	0.810	3,168	107.5	3,387	114.9	3,478	118.0	120.1	(117.0)
3,516	4,387	1.321	3,986	90.9	(4,060)	92.5	(4,150)	94.6	(95.7)	(95.7)
(7,270)	8,539	1.384	8,052	94.3	(8,200)	96.0	(8,400)	98.4	(99.5)	(99.5)
(9,553)	11,530	1.537	(10,874)	94.3	(11,000)	95.4	(11,700)	101.5	(104.1)	(102.3)
35,020	23,907	0.836	25,929	108.5	(27,000)	112.9	(27,000)	112.9	(112.1)	(112.1)
(18,000)	(19,820)	1.541	(20,128)	101.6	20,579	103.8	21,584	108.9	108.4	108.8
4,647	4,229	1.393	4,120	97.4	4,183	98.9	4,257	100.7	100.2	99.8
5,447	5,240	2.145	5,431	103.6	5,280	100.8	5,669	108.2	110.4	112.7
4,303	4,283	1.470	4,316	100.8	(4,350)	101.6	(4,400)	102.7	(102.7)	(103.9)
4,633	4,215	1.834	4,656	110.5	4,477	106.2	4,791	113.7	(112.5)	(113.9)
35,720	34,566	1.615	35,283	102.1	34,762	100.6	36,702	106.2	105.2	105.9
49,600	49,192	1.597	51,506	104.7	50,604	102.9	53,443	108.6	106.8	107.7
58,330	59,290	1.539	63,205	106.6	62,597	105.6	66,258	111.8	108.9	109.6
4,819	2,910	0.201	2,941	101.1	3,197	109.9	3,663	125.9	(146.0)	(140.7)
8,218	4,910	0.133	4,906	99.9	5,346	108.9	6,492	132.2	158.3	157.3
30,390	21,154	0.337	19,947	94.3	(21,150)	100.0	(23,000)	108.7	(114.6)	(114.6)
49,770	36,200	0.409	35,845	99.0	37,734	104.2	39,566	109.3	114.5	114.0
65,520	48,380	0.407	48,218	99.7	50,352	104.1	51,880	107.2	112.3	112.8
(99,000)	(80,000)	0.471	84,844	106.1	93,130	116.4	93,900	117.4	120.9	120.9
156,800	134,520	0.192	145,670	108.3	(165,500)	123.0	(168,200)	125.0	(129.0)	(127.8)
445,500	413,550	0.443	426,000	103.0	(460,000)	111.2	(472.000)	114.1	(114.9)	(115.3)
550,700	500,140	0.442	514,400	102.9	555,800	111.1	574,870	114.9	(116.5)	(117.3)
573,200)	(523,670)	0.456	(539,600)	103.0	(567,100)	108.3	(579,000)	110.6	(110.8)	(110.7)
668,000)	(581,870)	0.468	(596,085)	102.4	(629,200)	108.1	(649,800)	111.7	(110.7)	(110.9)
1,939	1,070	0.073	994	92.9	(1,160)	108.4	(1,410)	131.8	(179.0)	(186.9)
4,710	3,351	0.565	3,045	90.9	3,079	91.9	3,154	94.1	98.5	96.7
7,094	4,842	0.865	4,233	87.4	4,396	90.8	4,588	94.8	98.4	(97.4)
11,380	8,247	0.801	7,983	96.8	8,388	101.7	8,487	102.9	104.9	(105.4)
6,855	4,502	0.517	4,675	103.8	5,169	114.8	5,331	118.4	120.1	(120.1)
5,139	4,717	0.378	5,372	113.9	5,648	119.7	5,702	120.9	123.8	(120.8)
3,675	3,522	0.253	4,007	113.8	4,035	114.6	4,400	124.9	134.0	(131.6)
17,410	15,250	1.131	16,405	107.6	17,617	115.5	17,158	112.5	109.5	108.4
6,010	4,608	0.187	5,122	111.2	6,232	135.2	6,496	141.0	155.8	165.3
56,640	53,300	0.101	59,520	111.7	71,257	133.7	73,165	137.3	142.0	138.9
8,303	8,211	0.587	7,527	91.7	9,285	113.1	(9,600)	116.9	(121.8)	(125.4)
(37,000)	(28,674)	1.112	30,250	105.5	(32,000)	111.6	(30,000)	104.6	(106.4)	(104.6)
46,590	36,088	0.229	36,200	100.3	40,260	111.5	41,240	114.3	110.8	103.1
(11,240)	(9,818)	1.318	10,293	104.8	(10,500)	106.9	(10,500)	106.9	(101.8)	(106.9)
(50,000)	(30,060)	0.456	30,120	100.2	29,090	96.8	30,630	101.9	97.0	91.6

Red River Landing, La. [e] Gage heights recorded at a neighboring station.

TABLE 8—(*Continued*)

No.	Mean Discharge and Percentages (*Continued*)						Maximum Discharge			
	Years of Record (*Continued*)						Q (max)		Date	Myers rating
	40	50	60	65	80	100				
	19 %	20 %	21 %	22 %	23 %	24 %	25 cfs	26 csm	27	28 %
3	(107.0)	(108.0)	(107.0)	(106.5)	(106.0)	(105.0)	196,000	32.8	3–17–1865	25.4
4	(95.3)	(97.8)	(99.1)	(98.4)	(97.5)	(96.2)	200,000	116.	3–18–1936	48.3
5	(99.3)	(100.1)	(102.5)	(102.0)	(101.6)	(99.7)	74,300	57.0	7–25–1912	20.3
6	(100.7)	(98.7)	(98.1)	(97.4)	(96.1)	(94.8)	160,000	118.	7–10–1888	43.4
7	(98.8)	(99.2)	(100.0)	(100.0)	(99.2)	(98.0)	92,500	69.8	3–18–1936	25.4
8	(101.6)	(102.0)	(101.6)	(101.3)	(101.0)	(100.0)	156,000	29.9	7–11–1888	21.6
9	(100.3)	(100.7)	(101.6)	(101.6)	(100.0)	(98.4)	574,000	29.4	3–18–1936	41.1
10	(102.1)	(102.9)	(104.0)	(103.7)	(105.0)	(104.0)	507,000	20.4	2 — 1884	32.2
11	(100.4)	(103.2)	(104.2)	(103.8)	(103.9)	(102.6)	(600,000)	15.8	3–30–1913	30.8
12	(99.8)	(103.6)	(104.2)	(103.8)	(102.9)	(101.6)	654,000	11.8	1–28–1937	27.8
13	(101.1)	(104.1)	(104.9)	(104.1)	(103.1)	(102.1)	894,000	11.6	1–26–1937	32.3
14	(101.5)	(104.3)	(105.2)	(104.3)	(103.5)	(102.6)	1,110,000	12.2	1–26–1937	36.4
15	(101.2)	(103.7)	(105.8)	(104.4)	(104.0)	(105.0)	1,410,000	13.2	1–29–1937	43.1
16	105.5	(103.9)	(107.4)	(105.9)	(107.1)	(111.0)	1,850,000	9.1	2– 1–1937	41.0
17	(100.0)	(103.0)	(104.0)	(103.0)	(103.8)	(103.0)	270,000	36.4	3–27–1913	31.4
18	100.1	103.7	104.2	(102.4)	(103.2)	(102.4)	270,000	32.3	9–14–1878	29.5
19	(107.7)	(112.0)	(113.0)	(114.0)	(111.0)	(110.0)	260,000	67.6	3–26–1913	42.0
20	(108.5)	(113.6)	(115.3)	(117.0)	(113.6)	(111.9)	352,000	96.7	3–26–1913	58.4
21	(98.0)	(100.3)	(103.9)	(104.9)			86,200	26.0	1–23–1937	14.9
22	(101.9)	(104.2)	(107.7)	(108.0)			99,000	16.0	1–24–1937	12.6
23	(103.2)	(104.1)	(107.5)	(108.0)			208,000	27.7	1–27–1937	24.0
24	(108.8)	(110.8)	(112.9)	(112.1)	(112.9)	(117.1)	428,000	15.0	3–30–1913	25.3
25	105.9	105.0	(104.6)	(104.4)	(103.9)	(106.0)	203,000	15.8	1– 1–1927	17.9
26	(106.4)	(106.4)	(111.1)	(108.8)	(109.9)	110.8	(200,000)	65.9	3–10–1867	36.3
27	(114.5)	(114.5)	(115.8)	(116.4)			(137,000)	56.1	3 — 1867	27.8
28	(105.1)	(106.2)	(108.6)	(109.7)			70,000	24.0	3–23–1929	13.0
29	(118.6)	(118.6)	(121.0)	(122.2)			70,000	30.5	3–31–1886	14.6
30	107.4	107.9	110.2	110.9	(109.9)	(112.8)	459,000	21.4	3–11–1867	31.4
31	107.3	(107.7)	109.8	110.8	(109.8)	(113.8)	444,000	14.4	3–19–1897	25.3
32	108.0	107.3	(109.6)	(110.5)	(109.6)	(113.0)	460,000	11.9	3–24–1897	23.4
33	(154.5)	(159.6)	(164.9)	(161.5)	(160.0)	(156.0)	31,300	2.2	4– 5–1917	2.6
34	179.4	(181.3)	(190.0)	(185.3)	(183.3)	(177.2)	107,000	2.9	4–29–1881	5.6
35	(129.1)	(127.6)	(130.0)	(129.1)	(129.1)	(127.6)	(160,000)	2.6	6–19–1880	6.4
36	124.3	123.5	130.8	132.1	(132,6)	(129.8)	250,000	2.8	6–25–1880	8.4
37	120.5	118.1	125.6	(126.9)	(128.2)	(126.1)	314,000	2.6	5–18–1888	9.1
38	126.6	(123.8)	(130.0)	(131.3)	(132.5)	(130.0)	(350,000)	2.1	6 — 1844	8.5
39	(131.9)	(128.0)	(133.3)	(133.3)	(131.0)	(128.8)	1,300,000	1.85	6–28–1844	15.5
40	(116.1)	(113.2)	(117.0)	(116.1)	(115.8)	(116.6)	2,520,000	2.70	2– 9–1937	26.1
41	(117.6)	(115.0)	(115.2)	(114.4)	(114.0)	(117.0)	2,160,000	1.91	2–16–1937	20.3
42	(111.2)	(110.1)	(110.4)	(110.8)	(111.9)	(115.7)	2,050,000	1.78	2–19–1937	19.1
43	(111.0)	(109.6)	(110.0)	(110.3)	(111.4)	(114.8)	(2,620,000)	2.11	2–18–1937	23.5
44	(200.0)	(196.3)	(215.0)	(205.6)	(200.0)	(196.0)	65,000	4.45	4 — 1881	5.4
45	(103.3)	(104.4)	(107.4)	(107.4)	(106.0)	(104.0)	35,800	6.04	3–26–1920	4.6
46	(105.0)	(106.0)	(112.0)	(112.0)	(110.0)	(108.0)	78,000	13.9	3–27–1920	10.4
47	(111.0)	(110.0)	(115.0)	(115.2)	(113.0)	(115.0)	80,800	7.84	9–16–1938	8.0
48	(125.0)	(122.8)	(128.8)	(129.9)	(128.0)	(129.0)	51,000	5.86	2–22–1937	5.5
49	(124.0)	(121.9)	(127.2)	(129.3)	(126.0)	(128.0)	67,500	5.41	3–19–1929	8.0
50	(138.1)	(133.4)	(140.8)	(142.0)	(139.1)	(136.3)	97,000	6.98	6– 1–1903	8.2
51	(112.6)	(110.8)	(114.1)	(114.8)	(114.1)	(111.4)	58,300	4.32	10– 9–1926	5.0
52	173.7	179.7	182.6	(180.0)	(175.8)	(169.3)	140,000	5.69	6– 7–1908	8.9
53	140.5	(135.1)	(138.8)	(137.0)	(131.3)	(127.6)	(600,000)	1.14	6–19–1844	8.3
54	(131.3)	(137.6)	(146.1)	(143.7)	(140.0)	(137.0)	150,000	10.7	6 — 1844	12.7
55	(104.6)	(108.1)	(101.1)	(104.6)	(101.1)	(101.1)	440,000	17.1	4–23–1927	27.4
56	104.2	(110.8)	(99.8)	(105.3)	(104.2)	(102.5)	422,000	2.67	6–22–1935	10.6
57	(96.8)	(96.8)	(96.8)	(99.8)	(99.8)	(98.8)	72,900	9.8	1–29–1932	8.4
58	96.1	(99.8)	(96.5)	(96.5)	(99.8)	(103.1)	210,000	3.2	7– 2–1908	8.2

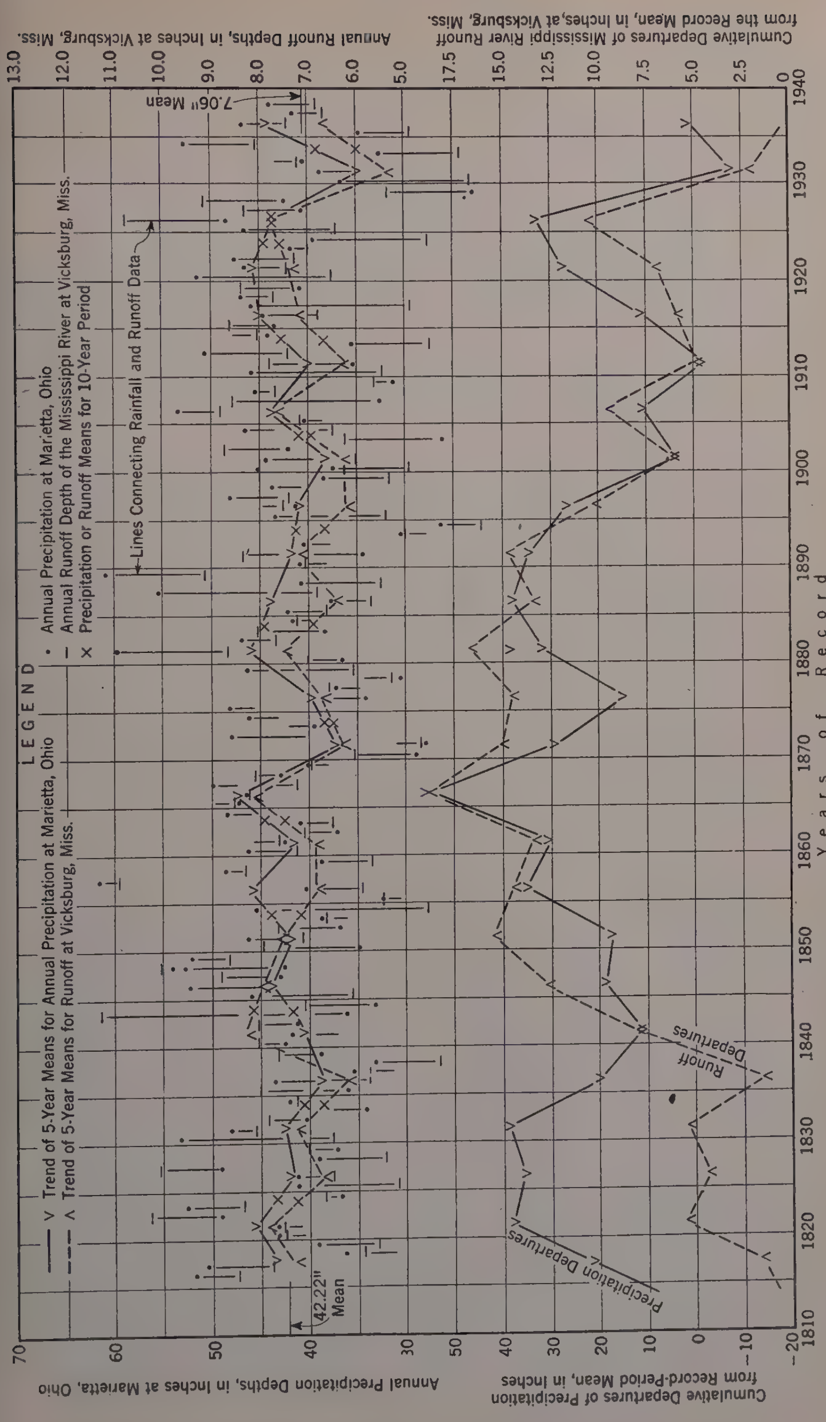


FIG. 4.—RELATIONS OF ANNUAL PRECIPITATION AT MARIETTA, OHIO, TO MISSISSIPPI RIVER DISCHARGE AT VICKSBURG, MISS.

Table 8 includes some of the most outstanding records of stream discharge to be found within the Mississippi Basin, either because of length of actual record of stream flow or of gage heights, or their association with other station-records from which extensions may be made with some assurance of their reliability. The mean discharges for various periods of record are expressed as percentages of the 5-yr mean, 1934–1938, which includes one year of very deficient rainfall and runoff, and one or more years of high floods.

The discharge in cubic feet per second for each of the 5 years included in the latest period there considered, the 5-yr mean in cubic feet per second per square mile, and the cubic feet per second quantities up to 30 years are all included, so that comparisons may be made conveniently among the various station records.

Like data are available for all of the important stations within the Mississippi Basin (more than 400 in all) dealing with graphic interpretation of hydrologic data in relation to land use.

To cover the assignment as outlined, it was found necessary to make evaluations of actual stream discharge, both for peaks and for monthly, yearly, and total record means. The inequalities encountered in the lengths of various station records led to the comparison, segment by segment, and to the extension of the shorter ones with the aid of neighboring station data. Thereby the records of the principal tributaries as well as the main river stations have been extended somewhat beyond the periods of previous evaluation, but generally not far beyond the period for which either gage readings or actual discharge determinations were available, in the vicinity.

If the methods used in making these compilations are acceptable to the profession, or if such modifications are suggested as will make the results more nearly what is required, then a foundation will be available on which to assemble, compare, and interpret the land use information.

Acknowledgments

Acknowledgments are due the office of the chief of engineers and also of the Mississippi River Commission for data supplied either in published or unpublished form, with permission to use such information; the chief of engineers, Maj.-Gen. Julian L. Schley, M. Am. Soc. C. E., for his review of the basic methods used and for valuable suggestions; members of the U. S. Geological Survey for great volumes of required information, and particularly to N. C. Grover and R. W. Davenport, Members, Am. Soc. C. E., and G. C. Stevens. Acknowledgment is also made to members of other federal departments and bureaus dealing with basic data, particularly the U. S. Weather Bureau, the Soil Conservation Service, the International Boundary Commission and its engineer, Karl F. Keeler, M. Am. Soc. C. E.; Maj. Harry Larson, Assoc. M. Am. Soc. C. E., Engineers' Reserve, for the use of his 40-yr discharge determinations on the Lower Mississippi, incorporating many helpful suggestions and technical direction of the late Floyd A. Nagler, M. Am. Soc. C. E.; and the following members of the Soil Conservation Service: C. E. Ramser, M. Am. Soc. C. E., and R. S. Goodridge, Assoc. M. Am. Soc. C. E., also R. L. Stevens, Assoc. M. Am. Soc. C. E., and H. C. Murto, for both assistance and encouragement in a tedious undertaking.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PILE-DRIVING FORMULAS

PROGRESS REPORT OF THE COMMITTEE ON THE BEARING VALUE OF PILE FOUNDATIONS

Discussion

BY C. W. DUNHAM, M. AM. SOC. C. E.

C. W. DUNHAM,[74] M. AM. SOC. C. E.[74a]—The Committee might be more specific in its statements regarding the kinds of materials in which the safe static loads on piles seem to have the most direct relationship to the resistance encountered in driving the piles. As the writer understands it, such cases are approximately as follows:

(1) Piles of moderate length driven through very soft materials to, and slightly into, sands and gravels that are not underlain by highly compressible soils, these piles being primarily end-bearing ones;

(2) Piles driven into deep beds of sand that are not underlain by highly compressible soils and that are not of a quicksand nature, these piles being largely of the friction type; and

(3) Piles driven through moderately thick layers of compressible soils and then slightly into a noncompressible stratum not underlain by other compressible strata, the piles offering resistance partly because of friction and partly because of end bearing, the conditions being such that the pile "fetches up" rather suddenly when driven, and such that a large proportion of the load can be carried by end bearing, if necessary, without objectionable settlement.

It is probably true that the preceding conditions are seldom serious for any reasonably good pile foundation and, if so, they can be treated separately from the dangerous conditions with, perhaps, beneficial emphasis upon the latter.

NOTE.—This Report was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by Messrs. G. G. Greulich, C. O. Emerson and D. O. Northrup, Harry J. Engel, and John D. Watson; October, 1941, by Messrs. Robert D. Chellis, Lazarus White, John G. Mason, Carlton S. Proctor, George Paaswell, and Abraham Woolf; November, 1941, by Messrs. Howard T. Evans, William G. Atwood, Donald M. Burmister, Wallace E. Belcher, Clement C. Williams, and D. P. Krynine; December, 1941, by Messrs. Trent R. Dames and William W. Moore, Maxwell M. Upson, Gregory P. Tschebotarioff, Robert F. Legget, and Jacob Feld; January, 1942, by Messrs. Lewis C. Wilcoxen, H. A. Mohr, and A. E. Cummings; and February, 1942, by Messrs. Karl Terzaghi, Ralph B. Peck, and Arthur Casagrande.

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[74a] Received by the Secretary February 13, 1942.

At any rate, a pile-driving formula for such cases may be used with some degree of confidence, provided that (as Hardy Cross, M. Am. Soc. C. E., has often stated) experience has shown that the results have given satisfactory answers to the questions: "Is it safe? Is it wasteful?"

A formula for such cases should be simple and easily used. One which depends upon various and variable coefficients, whose values are subject to guessing and change without notice, is confusing and deluding. Every one agrees that the results obtained from such a formula are not correct but, if they are reasonably so and moderately conservative, one may as well arrive at the results simply rather than through devious mathematical procedures whose greater value is probably psychological rather than real. Whether or not the "Engineering News" formula is satisfactory is a question. In any case, it has an extensive and valuable history of use from which engineers can obtain a conclusion regarding its acceptability under the foregoing conditions.

In the cases of pile foundations in deep strata of compressible soils, or when underlain by such soils, there is real danger of objectionable settlement. The general public, as well as engineers, should be warned of this emphatically with the hope that owners and financial backers of projects will authorize adequate investigations and tests of conditions at each site, prior to completion of the design, if possible, but certainly before the letting of contracts and the starting of construction. They should be warned also that there is no good substitute for the knowledge and judgment of capable engineers who have had adequate experience in such foundation problems.

However, the Committee is faced with the question of what to advise engineers to do when designing and building small structures on compressible soils for which suitable tests are not, or cannot be, made in advance. The designer must proceed with something, and the contractor needs specific data upon which to bid. If the Committee gives no pile-driving formula, the engineers concerned with the job will almost surely decide upon a safe load for the piles, and then they will select a formula of their own choosing to guide them, striving to realize the hoped-for bearing capacity, meanwhile feeling that the Committee has left them adrift.

Rather than this, it seems desirable for the Committee to adopt a pile-driving formula even for these cases, poor though it may be. With little possibility of being correct anyway, the formula should be simple. Too complicated a formula may induce people to think that it really is reliable. Then the Committee should state in the strongest possible language that, denied the assistance of proper tests in advance, the designer must do the following:

(1) Accept no alternative to a course of conservatism because he must deal with factors that affect the safety of life and property; and

(2) Let the hidden costs due to this conservatism be a financial burden carried unconsciously by the owner because of the lack of adequate information.

Correction: In February, 1942, *Proceedings*, page 321, transpose Figs. 13 and 14.

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DISCUSSIONS

FUNDAMENTAL ASPECTS OF THE DEPRECIATION PROBLEM A SYMPOSIUM

Discussion

BY MESSRS. THOMAS R. AGG, CONDE B. MCCULLOUGH, W. V. BURNELL, ROGER L. MORRISON, WALLACE B. CARR, W. L. WATERS, A. G. MOTT, DAVID A. KOSH, E. G. WALKER, AND K. LEE HYDER

THOMAS R. AGG,[33] M. AM. SOC. C. E.[33a]—The several papers presented in the Symposium on depreciation give the impression that one may consider that there are several depreciations existing in a property at any time and that one may select the particular depreciation that is applicable to the purposes of a particular analysis. This is a fundamental misconception that has retarded greatly the correct handling of the loss of value of physical assets through age and use in valuation and accounting.

Perhaps the confusion that is so apparent has its origin in the fact that the word "depreciation" (*de* = down + *pretium* = price) has a definitely qualitative meaning in the strict dictionary sense but over the years has gradually come to be used rather in a quantitative sense. One of the first steps toward clarification of the subject would seem to be to adopt suitable modifying terms to use in conjunction with the word when it is used in a quantitative sense to indicate the dollar equivalent of the decrease in future usefulness due to age and use, including functional depreciation in so far as it can be estimated. The following may serve to indicate a step toward the clarification of the use of terms.

Actual depreciation (annual or total) is the true loss of value for the period considered, determined by estimating the fair value at the beginning and end of the period. This is the one true figure on depreciation and the one that

NOTE.—This Symposium was published in November, 1941, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: November, 1941, by Messrs. Edwin F. Wendt and L. T. Fleming, and Anson Marston; December, 1941, by Messrs. William G. Atwood, George E. Goldthwaite, Nathan B. Jacobs, H. J. Flagg, Nelson Lee Smith, and C. Beverley Benson; and February, 1942, by Messrs. Paul T. Norton, Jr., H. L. Ripley, and Carroll A. Farwell.

[33] Dean of Eng., Iowa State College, Ames, Iowa.

[33a] Received by the Secretary January 8, 1942.

must be used when the correct annual or total depreciation reserve accounts are being set up and when deductions are being claimed for tax purposes. It is the figure that represents the true loss in value of the property from all predictable causes and, were it possible to overcome the irrational use of the term "depreciation," it might be wise to adopt this conception as the definition of the term for the purpose of engineering economics.

Theoretical depreciation (annual or total) is an estimate of the loss of value, during the period considered, which is made according to some formula that does not require the determination of fair value at the beginning and end of the period. Theoretical depreciation may be less or greater than actual depreciation. It might, by merest accident, be equal to actual depreciation in certain cases. The use of theoretical depreciation only, in the management of properties, may lead eventually to financial difficulties.

A point has been made of the difficulty, in a year of low business activity, of providing depreciation appropriations from earnings that will be equal to the true depreciation of the property during that year. Good management would seem to require that the true actual depreciation be known and recognized even then. In fact it must be known if the proper deductions are to be used in tax returns and in fixing the true value of the property. The annual depreciation appropriation can be made according to the exigencies of the business situation, except for businesses whose rates are under public regulation, in which case an annual depreciation appropriation equal to the annual actual depreciation should be made before calculating and distributing the net return.

The several papers make frequent reference to the difficulty of forecasting the future service life of a unit of industrial property because of the impact of obsolescence, inadequacy, and similar elements of functional depreciation. This is a bogey that has long confronted the appraiser, but in reality such factors, when actually unforeseeable by qualified experts, have nothing whatever to do with the problem.

The value of an asset as of today is the present worth of its future operation returns. Since they lie in the future, these returns can only be forecast according to the best judgment of qualified persons; but once such a forecast is made and accepted by the management of an enterprise, it becomes the present value. Those things that cannot be foreseen cannot affect the value until the future has become the present and the now unforeseeable is actually foreseen.

The same confusion exists in connection with estimates of the probable future service life of units of industrial property. The probable future service life is estimated on the basis of various factors, such as:

(*a*) The present physical condition of the unit;
(*b*) The nature of the future service so far as that can be foreseen; and
(*c*) The mortality characteristics of similar property if they have been established.

Functional depreciation that cannot be foreseen at the time has nothing to do with the estimates of future service life; but established mortality data should, and usually do, include the effect of every factor that causes retirements, such as accidents, changes in the art, inadequacy, supersession, and all

like factors—that is, present value is an estimate and probable future service life is an estimate. The actual service life cannot be known until it is too late for the information to be of any use in dealing with the actual depreciation of that particular unit of property.

If several well-qualified persons make estimates of the actual depreciation of a property on the basis of their best judgment, and without bias, it is extremely unlikely that they will arrive at approximately identical figures. Try having three civil engineers make independent estimates of the value of a used transit, for example. There is nothing disconcerting in this situation, and the courts have long recognized it to be inevitable. All values rest on the forecasts of the future, and these must always be subject to the uncertainties that characterize the human mind when it seeks to penetrate the future. In engineering economics, however, there is no alternative to the use of the best estimates that can be made by those responsible, and such estimates of fair value and actual depreciation must be accepted for all administrative purposes until the shadows of coming events provide a basis for a re-estimate of the situation.

There are various philosophies about the economic considerations that are applicable to public expenditures for such things as highways, canals, harbors, power plants, dams, and the like. One is that such expenditures may be considered a governmental function just like those required for the courts, fire protection, sanitation, police protection, and similar familiar governmental activities. If that is accepted, including the principle that no particular consideration need be given to the cost of the service so long as there is no misappropriation of funds, then engineering economics has no place in the establishment of policies; but on that basis the costs will inevitably be very much higher than really required. If it is desired to furnish service at the lowest cost, all alternative methods of producing the service must be studied on the basis of the true cost, and this will inevitably require consideration of the actual depreciation to be expected. Such things as annual depreciation appropriations and depreciation reserves will not be required, however.

If the public utility theory of such improvements is preferred, the several facilities will be operated by the government on a nonprofit basis, but revenues in the form of taxes will be exacted in an amount equal to the annual cost of the service. Here again the principles of engineering economics, and especially those relating to depreciation, must be considered if the services are to be furnished at minimum cost.

In the long run the public will be served best if governmental agencies keep, and utilize intelligently, records that will enable them to determine correctly the annual cost of the service of the utility they manage for the public and to make periodic reports summarizing these data.

Conde B. McCullough,[34] M. Am. Soc. C. E.[34a]—The presentation by Messrs. Crum and Winfrey is timely and valuable. All too frequently have engineers failed to account correctly for depreciation of public properties.

[34] Asst. Chf. Engr., State Highway Dept., Salem, Ore.

[34a] Received by the Secretary January 20, 1942.

Furthermore, as these authors so aptly point out, there is apparent a regrettable lack of understanding on the part of many engineering administrators in regard to the real nature of depreciation as an element of cost.

Messrs. Crum and Winfrey state (see heading "Value and Depreciation: Definition of Depreciation"):

> "The depreciation of any physical property unit, at any service age, is its loss of value, since its service started, due to decrease in the present worth of its probable future earned operation returns, below what such present worth would be if the unit were still new."

This is a clear-cut definition and should serve to eliminate any misapprehension as to the nature of depreciation itself. It might be enlightening, however, to pursue this definition a little further in order to discover where it leads when reduced to mathematical terms.

Consider a given highway property unit whose first cost is C, and whose assumed service life is n years. Assume that the annual income from this unit will be uniform throughout its service life, and let such annual income be represented by the term I_a.

The "present worth of the probable future earned operation returns" of this property unit, as of the date when it was first put into service, is given by the expression

$$P_{no} = I_a \left[\frac{(1 + r)^n - 1}{r\,(1 + r)^n} \right] \qquad (2)$$

in which r is the rate of interest or net return. At the end of m years the present worth of the probable future returns from this same unit is lessened or depreciated by virtue of the fact that it has only $n - m$ years of service left. Applying the same formula, the present worth of these probable future returns at the end of the m-yr period will be given by the expression

$$P_{nm} = I_a \left[\frac{(1 + r)^{n-m} - 1}{r\,(1 + r)^{n-m}} \right] \qquad (3)$$

The depreciated value of this unit at the end of m years obviously may be obtained by multiplying its original cost C by the ratio, $\frac{\text{Eq. 3}}{\text{Eq. 2}}$, or

$$V_{Dm} = C \left[\frac{(1 + r)^{n-m} - 1}{r\,(1 + r)^{n-m}} \right] \left[\frac{r\,(1 + r)^n}{(1 + r)^n - 1} \right]$$

$$= C \left[\frac{(1 + r)^n - (1 + r)^m}{(1 + r)^n - 1} \right] \qquad (4)$$

in which V_{Dm} represents the depreciated value at the end of the mth year. If m is made equal to unity the expression for the first year's depreciation is

$$D_1 = C \left[\frac{r}{(1 + r)^n - 1} \right] \qquad (5)$$

The depreciation during any succeeding year (say the mth year) can be obtained from Eq. 5 by the formula

$$D_m = D_1 (1 + r)^{m-1} \qquad (6)$$

Clearly, therefore, the utilization of the definition of depreciation offered by Messrs. Crum and Winfrey (which, incidentally, appears to be the only logical and tenable one) develops a parabolic curve exactly coincident with the well-known curve for constant increment amortization over the same period. It will be observed that the annual depreciation varies from year to year. However, the sum of this depreciation and the interest on the depreciated balance does not vary but remains constant throughout the entire service period. It is given by the formula,

$$C_{ac} = C \left[r + \frac{r}{(1 + r)^n - 1} \right] \qquad (7)$$

This is a very important fact in that it indicates a method whereby the total capital cost (C_{ac}) of any project may be calculated by a single simple formula (Eq. 7) once the service life is determined and the interest rate established.

As shown by Messrs. Crum and Winfrey, the determination of the probable service life expectancy n of any individual highway property unit presents a problem difficult of exact solution. However, the work being done in connection with the various statewide highway planning surveys will eventually yield data from which this value can be estimated with some degree of certainty.

The term r is usually defined as the "fair rate of net return." Messrs. Marston and Agg define[7] this as "that rate which is just large enough to secure what capital is really needed for investment in enterprises of the particular character." In view of this definition the rate r is easily established as identical with the net interest rate fixed by the highway bond market as of the time of construction. With the foregoing data, therefore, it is only necessary to multiply the first cost of any highway property unit by the term $\left[r + \frac{r}{(1 + r)^n - 1} \right]$ in order to derive the annual capital cost—that is, the value of annual depreciation plus interest on undepreciated balances.

The foregoing derivation is based upon the hypothesis that the net rate of return will be uniform throughout the service life of the property unit in question. Such an assumption is contrary to fact in the case of many highway property units because of the possibility of changes in traffic density, or of a modification of rate structure by legislative fiat, and for other reasons. For the valuation of industrial property units it is often possible to take into account any modification in future operating return ratios; but for highway property units there appears to be insufficient data at present to support such a refinement in analysis.

The foregoing discussion is predicated upon the hypothesis of complete depreciation. In the case of highway property units, it is often possible to salvage a certain percentage of the original value for utilization in the recon-

[7] "Engineering Valuation," by Anson Marston and T. R. Agg, 1st Ed., 1936, McGraw-Hill Book Co., Inc., New York, N. Y.

struction. If it is possible to estimate the salvageable component V_s of any unit, the depreciation curve may be adjusted to take such fact into consideration, as indicated in Fig. 3. An inspection of this figure will indicate that, if

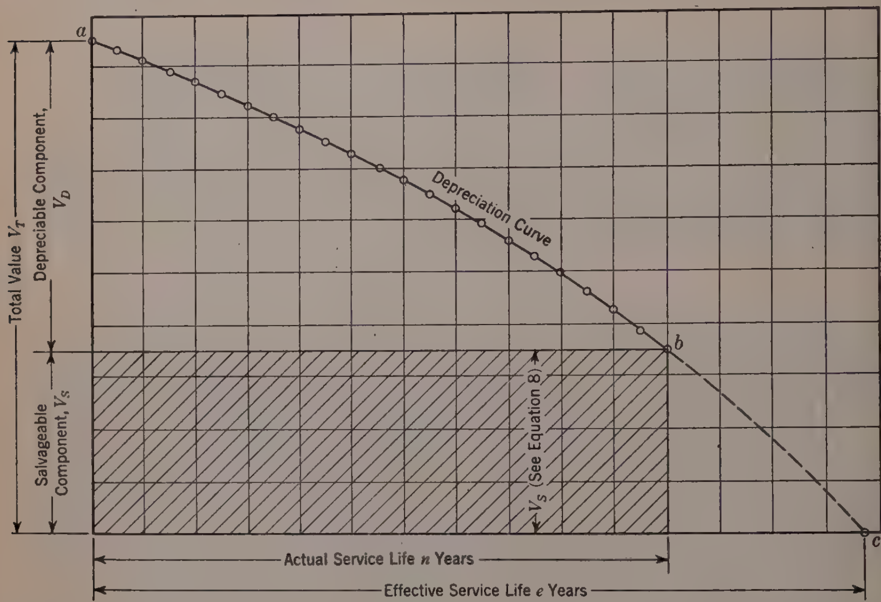

FIG. 3

the depreciation curve is prolonged beyond the point b to intersect the base line at point c, a new life value e is determined. From the relationships previously demonstrated for depreciation curves, in general, one may then write:

$$V_s = V_T \left[\frac{(1 + r)^e - (1 + r)^n}{(1 + r)^e - 1} \right] \quad \text{(8)}$$

from which:

$$(1 + r)^e = \left[\frac{(1 + r)^n - \dfrac{V_s}{V_T}}{1 - \dfrac{V_s}{V_T}} \right] \quad \text{(9a)}$$

Whence:

$$e \log (1 + r) = \log \left[\frac{(1 + r)^n - \dfrac{V_s}{V_T}}{1 - \dfrac{V_s}{V_T}} \right] \quad \text{(9b)}$$

and

$$e = \frac{\log \left[\dfrac{(1 + r)^n - \dfrac{V_s}{V_T}}{1 - \dfrac{V_s}{V_T}} \right]}{\log (1 + r)} \quad \text{(10)}$$

The computation of capital costs can then be handled by substituting the effective service life e for the actual service life n in Eq. 7.

The purpose of the foregoing discussion is simply to translate the Crum-Winfrey definition of depreciation into mathematical terms and to indicate that, if it is combined with the interest charge on undepreciated balances to form a total (which may be termed the capital cost), it can be computed by a single simple formula and will remain as a uniform charge throughout the years.

As stated, the only unknown quantity is the service life expectancy n or its equivalent e. In regard to highway property units, these data are being obtained in connection with the various state-wide planning surveys so that as the years pass it will be possible to make closer approximations for the initial setup. Moreover, since it is possible by means of Eqs. 2 to 10 to calculate residual values at the end of any given year, the value of n may be revised at any time during the service life of the property unit and depreciation schedules modified accordingly. Even though the initial assumptions are somewhat in error, it is possible, therefore, by means of frequent inspections and appraisals, to modify depreciation schedules so that they may be brought into closer coincidence with actual depreciation facts.

W. V. Burnell,[35] Esq.[35a]—The question of what to do about depreciation of public utility property, although it has been dealt with at great length and by many people, still remains a question. Is it necessary to do anything?

Irrespective of any requirement that has been or may be imposed by regulation, the fundamental answer is simply this: Prudence in business management requires that provision be made in advance for anticipated future losses—losses that occur when the productive usefulness of a property is lowered, and that culminate in the complete retirement of property from service when its usefulness has finally terminated. The making of such a provision is a process by which certain assets are earmarked as a reserve for this one purpose and no other—a process generally referred to as either retirement or depreciation accounting.

During the useful lifetime of a steam boiler, for example, the boiler itself is consumed, as definitely as the fuel that is burned in it; and this consumption of capital property is as truly a cost of producing steam as the cost of the fuel. If this process of retirement or depreciation accounting is to serve its purpose, the total cost of the boiler thus will eventually appear on the books of account as an expense of doing business.

At this early stage in the approach to the subject, a much debated question arises: "By what particular mathematical program should these retirement or depreciation charges be made?" Aside from the factor of regulatory compulsion, the question can never be answered categorically for all cases; each situation requires individual consideration in the light of its own surrounding conditions and circumstances.

Over the active life of a property a reserve must be built up, say from 0% at A (beginning of life) to 100% at B (end of life), as a cost of service. The

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[35a] Received by the Secretary January 22, 1942.

best method of laying out the path from A to B, or of allocating this cost of service, cannot be determined without consideration of the price at which the service can be sold. In a particular situation the path from A to B may best be that of a straight line; in another, a convex or a concave curve; in still another an irregular course, all depending upon the economics of the particular situation and the relative ability of the "traffic" to bear the charge at one time as compared with another. However, it always must be remembered that the total cost must be "set off" sooner or later against the total revenue. Where the responsibility for decision is left with those who have been entrusted with the management of a property, it may be suggested, as a general principle, that this cost of service may better be collected from the customer a little too early than too late—when the customer may decline to pay it.

No one, however, may properly conclude that business prudence invariably requires the rapid "writing off" of plant investment during the early years of its use. To do so, in many cases, would stifle expansion of the business by showing "losses" not in fact realized and, therefore, would be most imprudent. To have followed the "rapid-write-off" policy in the electric power industry, for example, would have resulted in a much higher "cost of service," with attendant higher rates, and thus would have discouraged the development of that industry.

From the viewpoint of basic principles, the foregoing statement is perhaps sufficient for present purposes because management is confronted with a condition, rather than a theory, with the artificial factor of regulatory requirements, and with certain extremely important corollary implications. This regulatory development, it is safe to say, is predicated on the underlying contention that management, or those who have been in control of management, in many cases have set aside insufficient reserves and, therefore, have failed to fulfil their responsibilities; that the reduction in the net book value of the property has not been enough to compensate for the depreciation actually suffered; and that the management has thus abused its privilege.

There can be no doubt that in many instances the accrued reserves are clearly too small and that dividend policies have often been too liberal, at the expense of the reserve. This is not to say, by any means, that a failure of the reserve to equal the amount determined by a particular accounting formula is sufficient justification for a conclusion as to the present adequacy of the reserve.

Management is now confronted, however, not so much by a theory as by a condition that imposes upon the custodians of industry an urgent obligation to contend vigorously and constructively for an equitable solution of the problem, which will definitely preclude the possibility of a confiscation of property. Otherwise, the advocates of a most unfavorable formula may succeed in carrying its application to the rate base and then "prove" that they are right by being in a position to enforce artificially a reduction in value which, as a matter of economics, has not occurred in fact.

Unless the management has labored long under a grievous misapprehension, the books of account of a public utility, like those of any business enterprise, do not purport to account for value, but for cost. In particular reference to

the depreciation reserve, if the management is correct, it is not within the province of the accountant to pass judgment as to the propriety of its size on the assumption that it is a function of value; but rather the accountant should regard it as a provision for eventual loss in terms of cost, and point out, purely as a matter of fact, the amount which the management has set aside, in anticipation of the loss.

A concept of depreciation accounting has been suggested, under which the reserve and the current appropriations to it would be determined on the basis of periodical engineering studies of losses actually realized, as opposed to the concept that the reserve should contain also a provision for future losses. Underlying this concept of accounting only for losses realized is the evident desire to harmonize the reserve with the amount of depreciation claimed in a rate case. Although this objective of "consistency," in the equity of the transaction as between the investor and the customer, is entitled to emphatic approval, there is a more practical and prudent way in which that objective can be attained.

If the contribution to the reserve actually made by the customer, as a recognized factor in the rates charged to him, is in excess of the deduction, if any, which is made in arriving at the rate base, let his current burden be reduced by an interest credit on the difference, at an interest rate commensurate with the characteristics of the reserve as a senior obligation. Thus, the requirement of "consistency" would be satisfied simply and effectually.

The next move is clearly up to the public utility industry. It must defend itself to be sure; but that is not all. It is not enough to oppose a program that is patently unfair. Its job is to propose a better one. Surely it is not constructive to attack all age-life programs on the mere ground that no one can predict, reliably, the life expectancy of utility property, for the most ardent proponents of such a program will admit the point and the corresponding propriety of a periodical adjustment of the estimates. Is it not a waste of valuable time, also, to argue for a return to the previously approved policy of retirement accounting, where a transition is now required to depreciation accounting? Would it not be a definite and constructive contribution to the solution of a serious problem if industry would adapt itself, as best it could, to regulatory preferences, so long as the principle of equity is not violated?

Any requirement that a reserve which was adequate under the previously authorized principles of accounting be suddenly built up out of surplus to meet the so-called deficiency should be "fought to the last ditch" and without compromise of principle, on the ground of the inequitable nature of such a requirement. From here on, however, why not meet the issue realistically? Develop an age-life program that will include a provision for the normal current requirements, plus an increment to recoup, gradually, the theoretical "deficiency" or, if the case requires, a minus increment to reduce an over-accrual, so as to bring the reserve to the theoretical requirement at some reasonable time in the future.

Management has available a wide range of latitude within which a specific program can be developed that will be the most suitable to the circumstances,

and also within the means, of almost any individual situation—all in accordance with the technical requirements of regulation and, which is more important, will accomplish the necessary transition in a manner that will be completely equitable to all the parties who have a legitimate interest in the problem.

ROGER L. MORRISON,[36] M. AM. SOC. C. E.[36a]—During the period when road construction activities were confined primarily to making main highways passable under all weather conditions, the economic advantages of such construction were too obvious to require much study. Under present conditions, however, with large expenditures being made and projected for the further improvement of roads that are already surfaced, economic factors must receive careful consideration if the funds are to be spent wisely. Depreciation, of course, is one of the most important of these economic factors and, as Messrs. Crum and Winfrey point out, it is one that has been largely ignored in the past, at least in accounting.

The definition of value used in this paper is "anticipated worth of future services" or "present worth of probable future service" (see heading "Value and Depreciation: Definition of Value"). One definition given for depreciation is "negative value." This does not seem to be quite consistent, because zero value would mean that future service is worth nothing at present, and negative value would seem to be a condition under which future service would be actually detrimental, as, for example, the present worth of the future "service" of termites working on a building. It is possible, of course, that an acceptable definition of "negative value" would make it synonymous with "loss of value." The point is of little importance anyway.

After the statement that "Depreciation is negative value * * *" appears the following (see heading "Value and Depreciation: Definition of Depreciation"):

> "The depreciation of any physical property unit, at any service age, is its loss of value, since its service started, due to decrease in the present worth of its probable future earned operation returns, below what such present worth would be if the unit were still new."

This covers total depreciation over any period from the beginning of service but omits a definition of depreciation in general. For instance, the depreciation of a property during its second five years of use could not be defined as "its loss of value since its service started" or as its loss of present worth, at the end of the ten years, compared to what it "would be if the unit were still new." A possible way to make the definition more general would be to substitute the clause "during any period of time" for "since its service started" and to substitute "below what its value was at the beginning of the period" for "below what such present worth would be if the unit were still new." Such a change would tend to complicate the definition, but the original does not seem to be exactly a "definition of depreciation."

The expression "probable future earned operation returns" is open to various interpretations when applied to a highway, and its interpretation is still

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[36a] Received by the Secretary January 22, 1942.

more difficult when applied to a schoolhouse or other public building. Perhaps in the latter case the earned operation returns would be the rent that would have to be paid for similar accommodations in a privately owned building.

In the case of highways, there is a tendency to divide the total vehicle taxes paid in a year by the total vehicle-miles traveled to determine the average tax per vehicle-mile, and then to compute the "earned operation returns" on any given road by multiplying the annual vehicle-miles traveled on the road by the average vehicle-mile tax. The writer objects to this because the motorists, or other taxpayers, finance road improvements in order to get returns in the form of reduced vehicle operating costs, or otherwise, and not to produce tax money for the states, counties, or other government units of highway administration. Considering the purposes for which the improvements are made, it would seem that their "earnings" are their returns to the motorists, or others, who pay for building them. The paper does not include an interpretation of earned operation returns.

The authors ask the question (see "Summary"), "Why * * * could not every state legislature, school board, and city council provide about a constant sum annually for building replacements and extensions?" One answer is that the needs are seldom constant. Population in any given locality may increase rapidly during one period and then increase slowly, or even decrease, during another period. Also, it is obviously much easier to raise funds for such purposes during prosperous times than during financial depressions.

Messrs. Crum and Winfrey have given extensive study to problems of highway economics and have done much to guide the economic thinking of highway engineers along sound lines. Their paper calls attention to points in regard to depreciation that will be of increasing importance if large sums are borrowed after the war and spent on highways primarily to furnish employment. It is under such circumstances that sound economics is most likely to give way to other considerations.

WALLACE B. CARR,[37] M. AM. SOC. C. E.[37a]—It is significant that the "Foreword" by Mr. Scharff and the papers by Mr. Walls and Professor Grant stress the state of confused thinking with which the question of depreciation continues to be surrounded. The Symposium is a decidedly progressive attempt to clarify this confusion.

Depreciation of property is essentially a term related to worth or capital represented by the property. This is indicated by its root derivation as well as by Professor Grant's three concepts of its meaning—"Decrease in Value," "Amortized Cost," and (the third one rephrased) Difference in Value of Two Assets Capable of the Same Service. This conception is also the basis of the definition written by Dean Marston in the paper by Messrs. Crum and Winfrey (see heading "Acknowledgment"). In all cases, however, depreciation should be determined factually or measured by the physical, functional, or economic ability to perform a prescribed or required service. The future anticipated period of such service is an important factor in the process of measurement.

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[37a] Received by the Secretary January 26, 1942.

This twofold significance of the term is responsible for much of the existing confusion. The factual measurement of depreciation, when expressed in terms of worth or capital, becomes a measure of the actual integrity of such capital. Accounting for depreciation, however, is generally treated as an amortization process, the rate of amortization being related to one of several methods of mathematical calculation and, in most cases, not to the facts of the case. Unfortunately, however, whatever process of mathematical calculation may be used, it is sometimes assumed that the results express depreciation correctly. Even though the period of amortization may be exactly equal to the useful life of the property, it does not follow that the factual depreciation has been equal, at all times, to that calculated by some mathematical law.

In discussing the problem of "Economy Studies for Proposed Retirements" in a competitive (unregulated) industry, Professor Grant states that "The only 'depreciation' involved is the difference between the net amount realizable from the immediate sale of the asset and the net amount realizable from its future sale." This statement may be open to question from a certain viewpoint. For instance, assuming a capitalistic economy, the objective or function of depreciation accounting with which this argument is really concerned could be regarded as the protection of the integrity of the invested capital. Another principle objective in this type of economy can be regarded similarly as the earning of a proper wage of capital while it is usefully employed. The accumulated accomplishment of the first objective would be reflected in the depreciation reserve, whereas that of the second objective would be reflected in the earned surplus account plus the accumulated payments of interest and dividends to the investors. Any deficiency of the reserve (which, according to Professor Grant's argument, should be charged to surplus) could not alter the fact that the reason for the charge would be depreciation. The procedure to record the facts properly would be to transfer the required amount from surplus to depreciation reserve and then to charge the retirement loss to the reserve. From this viewpoint, the answer would be simply that the reserve and surplus accounts had been stated improperly, rather than that the retirement loss should not be regarded as depreciation.

Carrying this viewpoint to a regulated industry, or to an industry subject for some period of time to excess profits taxes, the surplus might represent nothing more than the residue (after payments to investors), of a minimum, or even less than a fair, return. If in such a case the deficiency of the reserve for a retirement loss would not be chargeable as a depreciation expense in the future (which admittedly would mean a charge against the new asset), or if as a charge against present or future net income it would reduce that below a fair level, then one or the other of the fundamental objectives of a capitalistic economy would have been abandoned. Although it is true that in group-depreciation accounting this problem might not be of serious moment for the reason that under-accruals on short-lived items will be offset by over-accruals on long-lived items, nevertheless this situation should not obscure the fact that the objective of proper depreciation accounting should be the complete protection of the integrity of the invested capital.

Professor Grant's discussion of replacement valuation in appraisal matters is interesting. Theoretical economic support of this theory can be advanced, of course. It simply means that services at any time should be worth not more than the cost of producing such services by the most economical method. Practically, it may not be possible to achieve this ideal objective in all cases and, therefore, due regard to such a practical situation should be observed in the application of the theory in appraisals of existing property. If this theory were applied consistently to all enterprises in a given industry, and the market price of the product were based fairly upon the most economical production cost, the plants of higher production cost would meet this competition through the medium of depreciated plant values. However, it is questionable whether such a rigid rule can be invoked in the fixing of market prices or whether it is economically wise that it should be done. The question might be presented as to how to value, by this theory, a new plant of higher production cost than the most economical existing plant, the new plant being required to supply the market demand. Surely the new plant must not immediately, or even a year after completion, be depreciated to an "equal cost level" with the old plant. Rather, it might be that all the old plants would be appreciated to an "equal cost level" with the required new plant. Here again, as in all questions of value and depreciation, is a broad field for the application of sound engineering and business judgment. The effective leverage exerted by a small variation in the assumptions or estimates upon the final answer is well illustrated in Professor Grant's examples and should be a warning that judgment should be used generously in reaching a decision.

No discussion of depreciation can escape dealing with such terms as service lives, mortality curves or tables, etc. Mr. Walls discusses this phase of the problem very ably and emphasizes the character of the forces that control the useful life of physical property—in other words, the forces that cause depreciation. He states correctly that depreciation is of two kinds, physical and functional. The forces causing physical depreciation are wear and tear, decay, and normal action of the elements, all of which are controlled largely by natural laws and, therefore, operate quite continuously and with reasonable uniformity. Unlike these physical forces, those causing functional depreciation—inadequacy, obsolescence, changes in the art, and changes in demand and requirement of public authority—are essentially economic in character. They have no such natural uniform control, but rather they reflect the activities of mankind, such as his accomplishments in the field of invention, cycles and growth of business, movements and habits of people, etc. The incidence of their effect is erratic, therefore, and cannot be predicted correctly far in advance. It is folly to assume that mortality studies of historical data of capital retirements can always, or even generally, be used as a guide to future life with any hope of reasonable accuracy in the prediction. In the first place, mortality data of physical property generally reflect the composite effect of all causes of retirement, whether they are physical, functional, or a combination of both. Future retirements likewise will be made for various causes, some physical and others functional. In the past, functional causes have been a predominant element

in by far the largest number of retirements of property in regulated industry. The lives indicated by statistical analysis of mortality data as available from the past, therefore, are largely "functional" lives reflecting such economic or industrial conditions as were at work during the period of the data. To the extent that such statistically determined lives reflect the effect of these artificial forces of depreciation, the projection of these lives into the future is surrounded with a broad assumption of the continuity of largely artificial conditions.

If the causes of depreciation are thus differentiated, it then becomes apparent that physical property should likewise be classified as to its susceptibility to these various causes. This fundamental aspect of the problem has been brought out only indirectly in the Symposium. For instance, a large unit of property may have physically depreciable parts of such proportion and probable service life as to be equivalent from the viewpoint of turnover of capital to the complete depreciation of the entire unit in some period of time. If these physically depreciable parts are replaced continuously, as necessary, does physical depreciation proceed in the other parts of the unit in accordance with the equivalent over-all life, or are those parts immune to physical depreciation? Do they not await only the incidence of functional depreciation as, when, and if, it approaches in accordance with actual knowledge? If such a situation is true, progressive depreciation, calculated by some over-all formula or simply charged against property on a "hunch," would not represent factual depreciation because it would not be measured by a known impaired ability to perform a prescribed or required service. Rather, the calculation would represent amoritzation.

In general, the Symposium is sufficiently replete with warnings that judgment must be a very generous ingredient in any method of estimating or determining depreciation. The writer agrees heartily with Mr. Walls' inference that there is no substitute for sound engineering and business judgment.

W. L. WATERS,[38] M. AM. SOC. C. E.[38a]—Depreciation is the result of (1) wear and tear, and (2) obsolescence. In some cases, such as shoes, wear and tear is all-important. In others, such as women's hats, obsolescence is everything. Frequently, the average rate of wear and tear can be roughly approximated; but obsolescence is usually a pure guess, as it depends on the future progress of the art and the development of the property using the facilities. Therefore, the average over-all rate of depreciation to be allowed in any case usually cannot be anything more than an educated guess.

The Interstate Commerce Commission has had fifty years of experience with depreciation rates, so that its opinions can be considered authoritative. Its instructions are:[39]

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[38a] Received by the Secretary January 29, 1942.

[39] This quotation is from Section 8 of Special Instructions entitled "Depreciation of Fixed Improvements," on p. 33 of the Issue of 1914 of the I.C.C. "Classification of Operating Revenues and Operating Expenses of Steam Roads," to which Section reference is made in the various depreciation items in that Classification Order. This wording is repeated in Section 24, entitled "Depreciation Accounting," on p. 4 of the I.C.C. Order of June 13, 1934, entitled "In the Matter of Modifying the Uniform System of Accounts for Steam Railroads."

> "Depreciation charges shall be based in each instance upon the percentage of the original cost (estimated if not known), ledger value, or purchase price of the property determined to be equitable by the carrier's experience and the best sources of information as to the average current loss from depreciation."

This opinion indicates that the annual depreciation rate to be allowed is only an educated guess. It would be well for engineers to remember this, and not delude themselves with long-winded discussions that may only result in more muddle-headed concepts of the subject.

A. G. MOTT,[40] M. AM. SOC. C. E.[40a]—Professor Grant has contributed a remarkably well-stated paper on the subject of depreciation. It represents much progress of thought since the Final Report of the Special Committee of the Society to Formulate Principles and Methods for the Valuation of Railroad Property and Other Public Utilities published in 1917.[41]

In its broadest sense (and this is designated by Professor Grant as the "popular depreciation" concept), depreciation in its relation to value may be defined as loss of value from any cause. (Increase of value, or appreciation, may be properly included in this definition if recognized as negative depreciation.) To apply depreciation properly in the art of engineering appraisal, this broad definition requires a classification of causes. The various causes of depreciation appear to be fairly well agreed upon in the literature on the subject, and for convenience are listed herewith:

1. Physical causes, including wear, tear, decay, action of the elements, and the effect of casualties or accidents that have actually occurred prior to the time of appraisal (it will be noted that casualties or accidents that have not yet occurred cannot be included as a contributing cause of loss of value. In accounting practice, however, it is customary to determine the rate at which capital costs shall be spread over the period of use from an experience study of the average lives of the property. The effect of casualties is included in this experience. Depreciation reserves so accrued are in fact also insurance reserves);
2. Functional causes, including inadequacy, non-adaptability, changes of design, and other functional reasons for supersession;
3. Economic causes including the effect of legislative enactments, population changes, changes in habits, changes in the prices or supply of raw materials and labor, and changes in the market for the output; and
4. Monetary causes including the effect of changes in price levels, and other factors affecting costs of reproducing the property being appraised.

Since, by definition, depreciation is a loss of value, a proper application of depreciation can be made only after an understanding of the processes of arriving at value. If value can be ascertained directly at the beginning and at the end of the two periods for which depreciation is being considered, the

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[40a] Received by the Secretary January 30, 1942.

[41] *Transactions*, Am. Soc. C. E., Vol. LXXXI (December), 1917, p. 1448.

difference between the two values gives, directly, the amount of the depreciation; but, for appraisal purposes, it is a sum that has no significance, since the result (value) has already been obtained directly for the two periods. The usefulness of depreciation so obtained is to serve as a guide in predicting economic life or in forecasting future depreciation. Ordinarily, depreciation is merely one of the tools by which the appraiser finally arrives at value; and one of the misapplications of the use of this tool arises out of some confusion as to the nature of value itself. One of the most common errors in using the concept of depreciation is the attempt to use it as a device to convert, directly, some form of a cost figure to a value figure. This misuse of depreciation is excellently illustrated by Professor Grant's example of the $6,000 home bought in 1932 and sold in 1940 at the same price. One party to the transaction, translating original cost to present value, found no depreciation, whereas the other party, translating current reproduction cost to present value, found a $1,500 depreciation. (For the purposes of making this point, it is assumed that the sales price in this illustration is identical with the present value. The accrued depreciation found by the accounting process is $1,000, but this sum could not be subtracted from either original cost or reproduction cost to obtain value. The accounting process was for the purpose of spreading original cost over the life of the depreciable asset—a very different purpose from that of finding value during the useful life of the asset.)

In general, where the property being appraised is of a type that is being bought and sold frequently in a free and competitive market, the prices of such transactions may be conclusive evidence of value, and no determination of depreciation is necessary to arrive at value. The types of property which engineers are ordinarily called upon to appraise, however, are more commonly properties for which so few actual transactions take place that other tests must be applied to determine an appraisal which may be considered a fair substitute for market value.

The determination of value then depends upon the sound application of the appraiser's judgment to all the evidence of value that he is able to accumulate. Among the evidences of value more commonly considered are such factors as: Original cost of the property, the cost of reproducing the property new, the value indicated by a capitalization of earnings, and (for corporate properties) a value indicated by an appraisal of the stocks and bonds of the corporation. In a proper use of these or other factors, care must be taken so that any depreciation which properly applies to the particular factor is taken into account. (Depreciation is here used in a restricted sense; for example, cost figures are modified to express a comparison between the existing property and a similar new property to reflect physical and functional differences from a cost point of view; also the depreciation in the earning value is expressed by the difference in the present worth of the future estimated earnings for the existing property as compared with a new property.) By this means the appraiser can be more certain that he has in fact included somewhere in his appraisal all elements of depreciation in their proper perspective.

Of the factors mentioned two are definitely cost factors; namely, the original cost and the cost of reproduction. When depreciation is applied to these cost

factors, they remain nothing more or less than depreciated cost factors. (Professor Grant has well stated that those classifications of accounts which have defined depreciation in accounting as "loss in value" have contributed to confused thinking on the subject.) They have not become by that process a statement of value in themselves. The factor of earning value should have an allowance for depreciation. The proper earnings to capitalize are those that may be reasonably expected to be earned in the future after having allowed for all anticipated operating expenses, including the depreciation that will continue to accrue over the period for which the earnings are estimated. Obviously, the factor based upon the appraisal of the securities, properly arrived at in the first instance, would have reflected in it all forms of depreciation.

Another cause of confusion in the use of depreciation for appraisal purposes arises out of two different, but somewhat related, concepts for the term. These two concepts are depreciation as affecting value, and depreciation as a means of spreading the cost occasioned by the ultimate replacement or retirement of the property over the period during which the property was used.

The use of depreciation for value purposes is primarily an engineering and economic phenomenon, whereas the spreading of depreciation to current expense is partly engineering and partly economic, but largely an accounting and managerial function.

The application of age-life concepts of depreciation for value purposes should exclude the lives of those particular units which have already been retired and are not included in the property actually being appraised. On the other hand, in setting up rates of depreciation to include in operating expense, it is proper that the cost incurred because of the retirement of the short-lived items of property also be included. Since much of both the theory and practice of depreciation is related to time (age-life, for example), it follows that the time value of money (interest) must be a necessary element in its measurement. The value concept is based primarily upon an appraisal of the future usefulness of the property, whereas the depreciation expense concept is based upon a requirement to make good the total consumption of the depreciable assets, including those already retired.

The foregoing differences indicate some of the reasons why a properly accumulated reserve for depreciation (referred to by Professor Grant as "amortized cost") will not represent, accurately, the amount of depreciation that should be deducted, even from a cost figure, to reflect a properly depreciated cost figure for consideration as evidence of value.

David A. Kosh,[42] Jun. Am. Soc. C. E.[42a]—Depreciation is an economic problem, and only secondarily a technological one; and unless this is realized, much confusion may, and usually does, arise.

Messrs. Crum and Winfrey define depreciation of a unit (see "Value and Depreciation: Definition of Depreciation") as "its loss in value, since its service started, due to decrease in the present worth of its probable future earned operation returns, below what such present worth would be if the unit were

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[42a] Received by the Secretary February 5, 1942.

still new." This definition is very restrictive since it places major, if not sole emphasis on the technological aspect of depreciation. It implies that if the equipment were new, there would be no depreciation! A vital distinction must be made between "Maintaining the Plant," and "Maintaining the Investment." The former is primarily a technological problem—that of keeping the plant at its optimum condition of newness; but providing for depreciation must do far more than that. The value of the investment must be maintained, and not merely its physical manifestations. "Until financial reports take full account of all items of annual costs," write Messrs. Crum and Winfrey in the "Summary," "and account for investment in physical properties, they must be regarded as only partial truths." This is quite true, but "investment in physical properties" is not synonymous with "physical properties," and these concepts may not be used interchangeably. A further difficulty arises when the service life of a piece of equipment exceeds the duration of the accounting period. Hence, the appraiser must make some allocation to this "annual cost" on account of depreciation—the annual provision for depreciation. The basis for such allocation is the crux of the problem. Here Mr. Wall's suggestion in the last paragraph is most important:

> "The problem that lies before engineers seems to be rather definite and is certainly pressing: First, a sound basic concept or common understanding of the true meaning and scope of depreciation must be developed; second, an orderly and basically sound procedure for measuring depreciation must be formulated * * *."

(The writer however sees no reason why the problem belongs primarily to the engineer.)

Although the following discussion is centered about depreciation in the public utility industries, the principles apply to depreciation in industry in general, both privately and publicly owned. The generally accepted scope of depreciation (as well as a listing of the major causes thereof) has been stated as follows:[43]

> "Depreciation, as applied to electric plant, means the net loss in service value not restored by current maintenance, incurred in connection with the consumption or prospective retirement of electric plant in the course of service from causes which are known to be in current operation and against which the utility is not protected by insurance. Among the causes to be given consideration are wear and tear, decay, action of the elements, inadequacy, obsolescence, changes in the art, changes in demand and requirements of public authorities."

This is substantially the same as the definition used by the Federal Power Commission; and, although it was formally applied to electric utilities, it is entirely appropriate in the same sense to all types of plant. In the light of the foregoing definition, Messrs. Crum and Winfrey elaborate on only a minor part of depreciation, neglecting the vital portion. They state (see "Value and Depreciation: Definition of Depreciation"), "Thus, depreciation is just a question of values at different dates based on forecasted (probable) numbers of

[43] "Uniform System of Accounts for Electric Corporations," New York Public Service Comm., p. 2.

years of future service and probable values of the yearly services." The "probable values" depend on strictly non-technological factors; hence, the writer cannot agree with the contention (under the same heading) that "Such forecasts can be made correctly only by the sound judgments of impartial persons who are qualified to understand the mortality and other technical characteristics of different kinds of physical property units, and who personally examine the particular units whose depreciations are being determined." Messrs. Crum and Winfrey are concerned with technological depreciation only —wear, tear, decay, and possibly depreciation due to changes in the arts. It must be realized that it is obsolescence, and its usual antecedent, change in demand, that is responsible for from 70% to 75% of all electric utility depreciation. It was not technological depreciation that so worried the railroad industry but a short time ago; it was not the lack of newness of its equipment. It was rather the "economic depreciation" engendered by the business cycle, and the reduction in demand resulting from motor truck and pipe-line competition. A change in demand will leave a brand new trolley quite depreciated. Inflation might reduce, by a considerable extent, the previously calculated depreciation on an old hydro-plant.

So much for the annual provision for depreciation necessary to show the real annual costs. What of total accrued depreciation? Messrs. Crum and Winfrey state quite correctly (see "Distinction Between Depreciation and Maintenance") that "Depreciation losses of value * * * are something which cannot be made good in a physical way until new units are substituted for the depreciated units at the ends of their service lives." How do they propose to make good "losses of value" that cannot be made good in a "physical way"? They propose "* * * to create a depreciation reserve, which should be at all times just equal to the total accrued actual depreciations of all existing property units. In this way the correct annual cost is known and the depreciation reserve account shows the true loss in value of existing property, from which, of course, the present true value of existing property is determined." There are major objections to this proposal:

(1) The authors consider only physical property units, and further assume, quite correctly, that the sum of the depreciation of individual units is the "total accrued actual depreciation" in the plant. A new fleet of trolley cars with no market should, on the foregoing basis, have no depreciation reserve, should show no "loss in value," and should have a true value of presumably the cost of the new cars.

(2) If the words "total accrued actual depreciation" are not intended to restrict the reserve to physical depreciation, then it is submitted that a depreciation reserve adequate to maintain the investment can never, except by accident, measure the "true loss in value." It cannot be denied that physical depreciation is the more certain, and more easily predicted and measured, and that depreciation resulting from obsolescence and loss in demand cannot even be predicted with any kind of statistical certainty. Such depreciation may not even be classified appropriately as a risk, but rather as an uncertainty.

Safety factors are nothing new to the engineer. Hence, such a depreciation reserve must carry in it an item very like insurance, dependent not on pre-calculated risk probabilities, as is insurance, but on true uncertainties. Thus such a reserve would exceed the accrued depreciation by this "insurance" amount. This item would be in the nature of a prepayment by the customers for economic depreciation not yet matured, but as sure to mature as physical depreciation. Such a reserve certainly could not be construed as a proper deduction from any figure to yield present value. This prepayment, by consumers of the service, may in no case be considered as a disinvestment of the plant owners. However, the consumer would be entitled to a return on this prepayment, until the contingency which it anticipates actually matures. Thus a reserve, as proposed by Messrs. Crum and Winfrey, is inadequate to maintain the investment, whereas a reserve adequate to accomplish that purpose is not a suitable deduction for present value determination. This is the reason for the insistence that a clear distinction must be made between maintaining plant and maintaining investment.

Although some of the foregoing details are predicated on the assumption that the users of the service should pay its entire cost, the major portions of the thesis would apply equally well to such services as highways or public water systems. In the latter instances, the community as a whole is the consumer and hence must bear the cost. An unused road is just as depreciated as an unused manufactured gas plant. The depreciation has occurred and has been paid for in advance; and an accomplished fact must be recognized.

Professor Grant is correct in holding that, "If the 'decrease in value' shown by an appraisal based on the cost of reproducing the service with the most economical modern plant should be much greater than the accounting depreciation, one might reasonably infer that the accounting depreciation was inadequate" (see "A Suggestion Regarding Depreciation Accounting"). Since he considers the service, and not the original physical plant, the economic, as well as the technological causes of depreciation, have been included. Hence, Professor Grant's method constitutes a good check on how much depreciation has occurred; it is a good historical milepost. However, it is an inadequate solution for the real problem—how much depreciation is occurring, and (which is of even greater importance) how much will occur in the future. It is usually economically impracticable to collect past uncollected depreciation costs. When such an attempt is made in an active market, the development of the market is stifled. When depreciation, resulting from a reduced demand, is felt, it is then impossible to add extra depreciation charges without further diminishing a collapsing market. As a rule, a large portion of the depreciation reserve must be in the nature of a prepayment if the reserve is to serve its purpose. The sad effects of inadequate depreciation reserves, caused by a misunderstanding of the rôle of economic factors in depreciation, are clearly seen in the street railway industry, the interurban lines, and the domestic market of the manufactured gas industry—to mention but a few.

If it is argued that change in demand and obsolescence do not belong in the list of causes of depreciation, that the Federal Power Commission and the New

York Public Service Commission, among others, are in error in so listing them, that change in demand is a risk to be taken by the investor and hence not to be insured by the consumer, that the inclusion of that type of depreciation results in "unsocial pricing"—then one is on the threshold of a very wide field of discussion. May it merely be stated that the consumer ultimately must pay for full "depreciation" in any case—either in operating expenses above the line, or in an increased return to the investor below the line. Again the field is only collateral to the issue of depreciation, and will not be developed further. However, regulatory commissions do list loss in demand and obsolescence as causes of depreciation and, working, as they are, within the institutional framework of a defined economy, engineers and appraisers must also do so. A discussion of possible changes in this framework is clearly beyond the scope of this discussion.

To recapitulate, the major causes of depreciation, as now conceived, are economic; hence they cannot be detected by tools (accurate as they may be) designed to observe physical depreciation only. The annual cost of depreciation, and the reserve accumulated therefrom, must reflect these major economic causes. Since a sizable portion of such a reserve would represent a prepayment in anticipation of economic depreciation, the deduction of this reserve for purposes of rate-base determination is incorrect. Present value reflects past and matured depreciation; a future reserve must, of necessity, consider future, and as yet unmatured, depreciation.

E. G. Walker,[44] M. Am. Soc. C. E.[44a]—Consideration of depreciation and of amortization and obsolescence which, although they involve separate ideas, are related intimately to it, enter—or, perhaps it is more accurate to write, should enter—into a very wide range of activities. Possibly because of this condition, much confusion of thought has arisen. It is well, therefore, before attempting detailed studies of standard methods of practical application, to concentrate attention upon an endeavor to formulate the fundamental conditions from which the practice of making financial provision for conservation of capital sunk in wasting assets originates.

It is implied in the Symposium that attention is directed exclusively to such assets as are met with in engineering practice and constructional and manufacturing industries. The three papers of the Symposium give a good idea of how wide even this restricted range is. They are well selected to show that, as Mr. Walls states, "depreciation cannot be made to suit a definition, but rather the definition must truly express the actualities of depreciation" (see "Synopsis"). These actualities may differ widely in their incidence.

A primary difficulty is the laxity with which the terms depreciation, amortization, and obsolescence are used among men of affairs, and the confusion that exists between these terms and maintenance. Only quite recently the writer, in connection with certain problems of assessment of values of an industrial plant, was met with the objection that certain provisions for depreciation were unnecessary because the practice of the directors of the plant in

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[44a] Received by the Secretary February 10, 1942.

question had always been to spend considerable sums on maintaining it in the best possible condition. This is the confusion referred to in Professor Grant's paper between "as good as new" and "as valuable as ever." The difference between the two is obvious to most people, although it does not appear always to be clear to the so-called "businessmen" whose influences figure largely in the direction of many engineering and manufacturing concerns. It may be well to emphasize this difference to an even greater extent than it is in the papers under discussion.

Experience also shows that similar confusion exists as to the meanings of the terms amortization and obsolescence, although there should be no difficulty in appreciating clearly the differences between them and the term depreciation. For the benefit of those who confuse these terms it is suggested that a somewhat greater reference to them would be appropriate in any report of the Committee on Depreciation than has been made in the Symposium.

The authors give, in some detail, their opinions as to what constitutes depreciation. They differ from one another slightly but are united in making provision for flexibility in treatment. Thus, Professor Grant distinguishes three kinds of depreciation and applies them according to the financial point of view of the party concerned with the assessment. He uses the terms "popular depreciation," "accounting depreciation," and "appraisal depreciation," and explains clearly in his paper the differences between them. Are they all rightly defined as depreciation, however? The home builder in Example 1 sold his home, after eight years, at the price he paid for it. He lost no money on the deal; but, even if he had maintained the property well, its life had been reduced by eight years and there had been a depreciation. The accountant and the appraiser take note of these facts, but the layman does not, so that, in the end, the author offers three sums—nothing, $1,000, and $1,500, respectively—on an asset originally valued at $6,000. An aim of the discussion is to determine the possibilities of measuring depreciation by engineering and scientific processes. In this simple example there are three sums differing widely in their relation to the original price. They are all referred to as depreciations, and the methods of assessing them are all based on the logical reasonings set forth in the paper. The question arises, if this line of argument is accepted, whether the appraiser is following engineering and scientific processes or is being led away from them along a course of opportunism—of adapting the argument to the circumstances—which is in essence nonscientific and unsuited to bring about the conditions set as the objective.

The nature of depreciation prevents the possibility of reaching a state in which it can be calculated with the exactness with which interests and discounts are computed. However, it may be possible to obtain for general adoption some more clear-cut procedure to enable any particular case to be examined and a fair judgment expressed as to the correctness and adequacy of the depreciation provision. Dean Marston's statements under the heading "Value and Depreciation" in Messrs. Crum and Winfrey's paper are progressive steps to this end, and establish fundamental principles. It is generally accepted that the fair price of any article or service is that paid in an unre-

stricted transaction in the open market by a willing buyer to a willing seller. In such a deal the buyer naturally can have regard only for what profitable use he can make of his purchase, or, in the terms of the paper, the "present worth of probable future service." The difference between the present worths at the beginning and end of any period must be the true appreciation or depreciation of the asset over the period. This fundamental principle provides a true measure that can be applied to any case at any time to ascertain whether adequate financial provision has been made to meet the changes arising from the conditions—some very direct, some quite remote—to which the asset has been subjected.

The application of the principle is not simple because it may involve much that is not susceptible of straightforward calculation. It implies in most instances exercise of skilled judgment and the enunciation of considered opinion, and thereby introduces possibilities of considerable divergence. One has only to think of the numbers of technical cases that are argued daily before the courts of law to realize the extreme divergence of opinions put forward by men of equal experience and judgment on matters that they approach from opposed viewpoints. If engineers are to arrive at a generally acceptable basis for estimating depreciation, they must rid themselves of this condition. Some sacrifice of ideals may be necessary to achieve practical results.

If true application of the principle were possible, determination of depreciation would be largely a matter of periodical revaluation, and the variation of the periodical appropriations would be very different in different cases. Applied to the finances of certain classes of heavy civil engineering works, such as those which Messrs. Crum and Winfrey claim, rightly, should be—but by no means always are—subjected to adjustment from this point of view, there would be many cases in which little variation would be found from year to year. In others, such, for example, as plant installed in new and rapidly developing industries, there would be large and irregular variations, and the influence of possible obsolescence would be much greater. (This factor is not absent even from cases of the most uniform depreciation.) A man well versed in the technicalities of an industry is well equipped for assessing the probable future life of an item of plant in that industry, and is better able to estimate the chances of drastic changes in processes, markets, etc.; but even he cannot tell whether or not some new invention or idea may be developed at any moment that will alter completely the capabilities of his plant for effective manufacture and competition. This obsolescence is a vital and incalculable factor in depreciation. Calculations based on data as to realized or estimated lives must be adjusted to allow for these incalculable possibilities if effective financial provision is to be made. When obsolescence is an important factor, preliminary estimates and economy studies may be stultified and an ample basis afforded for writing off values quickly, irrespective of actual real depreciation.

The accountant's approach to the problem is not concerned with physical factors. He must see that adequate provision is made so that money may be available for replacements or reconstruction when it is needed. He must

maintain a fund that is adequate for meeting any demands likely to be made on it. As long as there are resources out of which to meet demands arising from depreciation, he has done all that is required of him. If he is an auditor, he must see that depreciation provisions are reasonable. From this outlook has developed the idea of Professor Grant's "accounting depreciation." It might be described not as a depreciation but as a provision for depreciation. It is derived usually by an arithmetical process based on original cost and deductions therefrom at fixed or varying rates. On a single item, such as the simple one used in Professor Grant's Example 1, it may result in a wide difference between the actual depreciation (the "appraisal depreciation") and the provision for depreciation (the "accounting depreciation"), but over the large number of items that combine to form a complete engineering entity there is a much greater possibility of averaging. Book figures for depreciation provisions should be considered in this manner. Individually they may not give an accurate view of the actual value of the item, but in the aggregate, if the finances are controlled properly, the discrepancy between the depreciation and the provision made in the books to meet it should be far less, proportionately. Obviously, for sound finance the provision must exceed the depreciation.

The writer believes there is much in favor of a standardization of the term "depreciation" to mean one thing and one thing only. The idea of depreciation should be based on the physical conceptions that determine the future possibilities of the working life of the asset. The reduction in the present worth of these physical conceptions over a period is the depreciation over that period, whether resulting from the normal shortening of the working life or from extraneous causes. How the depreciation is to be met is a separate problem. As, in general, periods of numbers of years are involved, in ordinary cases there is no urgent necessity to keep strict mathematical equality between depreciation and the provision made to meet it, although obviously the difference between them must be kept within proper limits. The former may be irregular in its increments and the latter (usually) regular. It is essential, however, that during the greater part of the time the provision be ahead of the depreciation if a sound financial position is to be maintained.

In Professor Grant's examples he introduces a point that has not been elaborated in the Symposium—namely, the increase of value of an asset over a period in spite of its physical deterioration. The circumstance, although unusual, arises sometimes in connection with items of engineering plant. It is likely to do so increasingly with the rising demands for such items, which are a feature of the present time as the result of war conditions. When a machine, purchased some years ago at a lower price, must be replaced at a price that may be several time that paid originally, it scarcely can be expected that the provision for depreciation made over a number of years, when there was little or no indication of this increase, will be adequate to meet more than a fraction of the new cost. The corresponding problem of insurance can be dealt with by an increase of coverage, but in extreme conditions the greatest practicable increase in the rate of provision may be far from sufficient to write up the fund to the new price. It would be of interest to have the authors' views on this problem.

K. Lee Hyder,[45] M. Am. Soc. C. E.[45a]—Writers on the subject of "depreciation" almost invariably preface their remarks with comments as to the state of confusion that appears to exist in the minds of those called upon to deal with the matter, either practically or academically. The several contributors to this Symposium support this observation in the presentation of their papers. They proceed, however, to make notable contributions which, in turn, should be of inestimable value in clarifying the basic thinking. By inference, at least, if not specifically or intentionally, almost the entire scope of controversial material has been brought forward in one or more of these papers.

Nevertheless, after a thorough review of these papers, the writer feels that there still remains a tendency upon the part of each to merge discussions of "concepts" with those of "procedures." The emphasis throughout seems to be directed primarily to the methods and to the variations in such methods for differing purposes. Perhaps a rearrangement of the material would go a long way toward clearing up the confusion. Professor Grant alone has introduced his treatment of the subject by a section entitled "Three Fundamental Depreciation Concepts," in which he clearly differentiates the three fundamental or foundational approaches, and makes reference to Professor Bonbright's great work, "Valuation of Property,"[4] in presenting these fundamentals as "concepts." In the writer's opinion, if these are to be so accepted (and he makes no point to the contrary), then they should in turn be predicated upon a still deeper foundational premise—that is, the character of the property subject to depreciation.

There is no fundamental difference in the causes that bring about the depreciation of assets, whether such assets are used in regulated industry, competitive industry, or in public works. Certainly there is no difference in the life expectancy of a boiler or generator, as such, simply because it may be a part of an electric-light system under regulation, a private manufacturing enterprise, or a municipally owned plant. It may be concluded, therefore, that the basic segregation in the Symposium is made in order to recognize the differing treatments to be accorded in the case of (1) a business monopoly, (2) a competitive operation, or (3) a politically controlled budget.

If the appraiser is really to get down to fundamentals, the property itself should first be subjected to clarification. To illustrate: Assume the extremes as reflected in five types of depreciable assets (or "properties"), each of which—under the realistic approach—obviously must be accorded a different treatment, as follows: (*a*) A leased fee in a parcel of improved real estate; (*b*) a complex industrial "plant"; (*c*) an automobile; (*d*) an oil well; and (*e*) a motion picture.

Any one of the foregoing constitutes a "property" and a "depreciable asset"; that is, it will at some future date be expected to reach the end of its useful life. Now, the character of each of the foregoing properties (or assets) is different. The "leased fee" (type *a*), for instance, has a known or agreed upon future life.

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[45a] Received by the Secretary February 19, 1942.

[4] "Valuation of Property," by J. C. Bonbright, McGraw-Hill Book Co., Inc., 1937. See particularly Chapter X.

The "property" itself is in reality a right to receive a stated sum of money for a definite number of years. In this property, depreciation actually accrues at a rate that may be charted in the form of a parabola. Very little will accrue the first year, and the sum will continue to increase each year thereafter to the end of the period. The accruals may be determined by simply taking the difference in the "present value" (or "present worth") of the annual income as of any date and the original "present value," using ordinary interest or discount tables for such computations.

In the "industrial plant" (type *b*) the appraiser is dealing with a complex assembly of items comprising the depreciable plant, each of which is subject to an individual life expectancy from both physical and functional factors. However, the enterprise as a whole is further subject to all of the risks and hazards of competition, economic and social changes, etc., and hence such factors may enter into the depreciation problem. The wide variation in the life expectancy of the component parts, for reasons which need not be discussed here, leads to the usual adoption of the "straight-line" basis as more realistically applicable.

The economic life of "an automobile" (type *c*), by both nature and custom, is heavily influenced by the factors of obsolescence. This influence starts quickly, and it is generally realized that the moment the new car is driven out of the salesroom it must be considered to have taken on a substantial "depreciation." Therefore, the "curve" of type *a* is reversed in character, and again—realistically—the "heavier" sums accrue during the earlier years.

The economic life of "an oil well" (type *d*) and its "curve" of realistic depreciation are similar to that of the automobile, although perhaps for more basic reasons. The gas pressure when a well is first opened serves to bring out the highest production of petroleum during the first year; and, as the gas pressure (and volume of oil) declines, the flow is less, and "depreciation" (depletion) is reduced accordingly. (The analogy here, of course, is that, as the deposit is depleted, the investment is proportionately depreciated.) The curve can be rather closely defined by the experience of other wells, and the engineer may make his estimates accordingly. Finally, the "motion picture" (type *e*) is also a "property" and an asset subject to depreciation. Keeping to actuality, the value (or cost, if the purposes so dictate) is found by experience to depreciate at perhaps 85% the first year, 10% the second year, and 5% the third year. Whatever is left after three years may be viewed in the same manner as "salvage" in other types of property after the end of the useful life.

With this initial distinction in the character of property defined, it would appear that the points raised and discussed in the papers could be developed with greater clarity. To a large extent (except for certain illustrations), the entire scope of the material presented by Mr. Walls, Professor Grant, and Mr. Crum and Professor Winfrey may be considered to be directed toward the classes of assets similar to type *b*—namely, the complex industrial plant.

Under such assumption, it must then be concluded that the discussion to follow should be so limited. If this is true, it would support the opinion that the material is to be considered primarily as directed toward the questions of

procedure under the hypotheses of (*A*) controlled industry, (*B*) competitive industry, and (*C*) public works.

Carried a step further, all of the authors, in one manner or another, present the causes of depreciation which the writer, for discussion purposes, elects to summarize as physical deterioration, functional obsolescence, and economic obsolescence. There appears to be no clear-cut distinction made between "functional obsolescence" and "economic obsolescence" and—again—if such were recognized it might well tend toward clarification. Realistically, the factors embraced in such causes differ in character and hence are subject to segregation. Practically, it may be difficult to draw a definite line of demarcation. A theory may be set up, however, and the appraiser may proceed logically from that point to establish the proof. In the regulated industry, all plant items are subject to "physical deterioration" in exactly the same manner as in the competitive industry or the municipally owned plant.

In like manner (although perhaps not to the same degree) this may be said of "functional obsolescence." (A power plant becomes outmoded, irrespective of the type of operating control.) However, the obsolescence of a plant occasioned by changing economic conditions is usually present to a much less degree in regulated than in competitive industry. The possession of a franchise, granting monopolistic rights to perform given services in a community, places the control of retirements much more strongly in the hands of the public utility operator than in the case of the competitive enterprise. Of course, the artificial gas company faces the threat of the encroachment of natural gas, but nevertheless the hazard is greatly circumscribed.

Furthermore, the municipal plant is rarely faced with economic obsolescence to any great degree, except as public opinion may bring pressure to bear upon the political unit in demanding major changes in entire systems. Mr. Crum and Professor Winfrey's excellent paper admits the inadequate recognition of depreciation in the case of public works—even as they so ably set forth the need for a system and an accounting practice no less complete than that essential to other types of ownership.

Analyzing the papers from a slightly different standpoint, one may clarify the subject by bringing the various procedures suggested under what is termed the "two-property concept," by which is meant the classification of the complex property as:

(1) Tangible assets (the physical facilities or "tools" devoted to the conducting of the enterprise); and
(2) Intangible assets (the business itself).

Now, the problem may be narrowed for any particular discussion. The nature of the business (property (2)) and its operation under or outside of public regulation, as the case may be, would be reflected in the intangible assets, but in reality would present factors that might well limit the use and life expectancy (and hence the amount of, and provision for, depreciation) of the tangible assets. It is this influence that might be embraced, under the theory suggested, in the category of "economic obsolescence."

If the foregoing discussion is a correct interpretation of the papers, then the entire problem presented in the Symposium might be summed up as follows:

> "What are the proper procedures in estimating accrued depreciation and providing for future depreciation for the depreciable tangible assets of complex industrial enterprises which are (1) under public regulation, (2) of competitive character, and (3) public owned?"

With a thoroughly developed exposition of the underlying concepts of "property" and "depreciation" as a foundational section, then the treatment of the principles and methods—whether for value estimates, rate making, or accounting, and each under various types of control—might be more effectively presented.

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DISCUSSIONS

DEVELOPMENT OF TRANSPORTATION IN THE UNITED STATES

Discussion

BY JOSEPH B. EASTMAN, ESQ.

JOSEPH B. EASTMAN,[14] ESQ.[14a]—Some of the problems treated by Mr. Teal are not quite as simple as they seem. For example, when it comes to determining the relative costs of the different forms of transportation, an over-all picture is afforded by comparing the rates that the public pays and adding something for what one may think is the public aid received; but, after all, the important point is to determine in what particular places each form of transportation can serve best—what particular kinds of hauling they can do better than some other kind of carrier. Now that involves quite a detailed study. There are not many short-cut methods that will yield that answer. It requires a long and very careful study, and the great difficulty is that the picture is changing all the time. It is not static, and that is one of the great difficulties that the President's new Board of Investigation and Research will encounter. That part of the Transportation Act of 1940 was really taken from a bill drawn by the writer in 1938 for the creation of a national transportation authority (Federal Transportation Authority), and at that time he had in mind a permanent agency that would be studying these questions all of the time—not necessarily making big reports on them, but applying pressure here and there, exerting influence, and offering advice where it is thought it would do the most good.

As a purely temporary organization, directed to make a comprehensive study of this question of relative costs, the President's Board will encounter some difficulties before it completes its work. Public aid to transportation is a problem that could be discussed all day and all night. The writer has produced four volumes on that subject. Railroad administrators probably do not disagree with the writer so much when it comes to the water carriers. Probably they do not disagree with him greatly about the air carriers or the railroads.

NOTE.—This paper by J. E. Teal, M. Am. Soc. C. E., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by Messrs. Fred Lavis, and William J. Wilgus.

[14] Director, Office of Defense Transportation, Washington, D. C.

[14a] Received by the Secretary January 19, 1942.

They do disagree most emphatically with respect to the highway carriers, and there are many questions which are not only questions of fact but questions of public policy.

The question of whether it is possible to reduce the number of trackage miles, for example, between Chicago, Ill., and Cincinnati, Ohio, is a very large one. There is no doubt whatever that if the railroad systems were operated as a unit and the traffic were directed over the best routes between the important points, considerable money could be saved. There are some objections to that possibility also. The four lines of railroad between Cincinnati and Chicago do not follow precisely the same routes. There are many places on each of those lines that are not served by the others, and people at those points, of course, would not appreciate it if their service were abandoned.

An elaborate study of this problem was made under the so-called "Prince plan," which would combine the railroads into seven systems—two in the East, two in the South, and three in the West—and change the routing of traffic, concentrating it in various ways to obtain large savings. A group of railroad engineers and traffic men investigated that plan for the writer when he was federal coordinator. Whereas authors of the plan put the savings at about \$700,000,000, that committee reduced it to something like \$250,000,000. There is no doubt whatever that, in connection with any such plan, a large volume of objection would arise from the public in various respects.

For example, such a plan would leave Baltimore, Md., served by one railroad system; New York, on the other hand, would be served by two; and Chicago would be served by at least four, and maybe five or six. That kind of an arrangement would cause considerable objection. People at points located on what would become secondary railway lines under such a plan as that would complain because they were removed from the main arteries of commerce. It is not a problem that can be dealt with simply without considering all the various angles. The writer feels that there are important savings that could be made through consolidation of the railroads, combined with coordination (that is, cooperative effort in places where their interests are in common, such as at terminals), and at the same time preserve and probably improve the service to the public; but as he found when he was federal coordinator, this is a very debatable subject.

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DISCUSSIONS

LATERAL STABILITY OF UNSYMMETRICAL I-BEAMS AND TRUSSES IN BENDING

Discussion

By H. N. Hill, Assoc. M. Am. Soc. C. E.

H. N. Hill,[11] Assoc. M. Am. Soc. C. E.[11a]—In 1937 the writer had occasion to solve the problem of the lateral buckling of an unsymmetrical I-beam subjected to pure bending, and derived an expression which differs somewhat from that determined by the author. The writer's solution can be expressed in the notation of the paper, if an additional term is introduced—"e," which is defined as the distance from the shear (or flexural) center to the centroid of the section. The centroid will lie between the shear center and the narrow flange. The term "e" will have a positive sign if the wide flange is in compression, and a negative sign if the narrow flange is in compression.

Using the disposition of axes as shown in Fig. 2 of the paper, the differential equations of equilibrium for lateral bending and twisting can be written: $E\,I\,\frac{d^2y}{dx^2} = -\,M\,\phi$ (see Eq. 8), and

$$G\,K\,\frac{d\phi}{dx} - E\,(I_1\,c^2_1 + I_2\,c^2_2)\,\frac{d^3\phi}{dx^3} = M\,\frac{d}{dx}\,(y - 2\,e\,\phi)\,\ldots\ldots\ldots(63)$$

In Eq. 63 the expression for the twisting moment is only approximately correct. It is applicable for unsymmetrical I-sections of the kind dealt with in this paper, and in which the two flange areas do not differ greatly. The unsymmetrical I-beams commonly encountered in structural fields are of this nature. A more exact expression for the twisting moment may be obtained by summing up the twisting moments produced by the longitudinal stress on each unit differential area of the cross section. The expression may be derived as follows, considering the z-axis as coinciding with the center line of the web, with the origin at the shear center, and assuming that the flanges are narrow compared to the depth

Note.—This paper by George Winter, Esq., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by Robert K. Schrader, Jun. Am. Soc. C. E.

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[11a] Received by the Secretary February 13, 1942.

of the beam:

$$M_T = \int_A \frac{M}{I_Y}(z - e)\, dA \left(\frac{dy}{dx} - z\frac{d\phi}{dx}\right) z \ldots\ldots\ldots\ldots (64)$$

in which I_Y = moment of inertia about the principal axis normal to the web. Expanding:

$$M_T = \frac{M}{I_Y}\left[\frac{dy}{dx}\left(\int_A z^2\, dA - e\int_A z\, dA\right) - \frac{d\phi}{dx}\left(\int_A z^3\, dA - e\int_A z^2\, dA\right)\right] \ldots (65)$$

in which

$$\int_A z\, dA = e\, A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (66a)$$

$$\int_A z^2\, dA = I_Y + e^2\, A \ldots\ldots\ldots\ldots\ldots\ldots\ldots (66b)$$

and

$$\int_A z^3\, dA = Z_Y + 3\, e\, I_Y + e^3\, A \ldots\ldots\ldots\ldots\ldots\ldots (66c)$$

In Eq. 66*c*, Z_Y is the third moment of the area about the principal axis normal to the web. Therefore,

$$M_T = M\frac{d}{dx}\left[y - \left(2\,e + \frac{Z_Y}{I_Y}\right)\phi\right] \ldots\ldots\ldots\ldots\ldots (67)$$

For an **I**-beam in which the two flange areas are not greatly different, it is evident that the centroid will not vary greatly from a point at half the depth, and the term $\frac{Z_Y}{I_Y}$ will be very small. The areas of the two flanges of an unsymmetrical **I**-beam may be the same and yet their lateral stiffnesses may differ greatly.

Differentiating Eq. 63 and substituting for $\frac{d^2y}{dx^2}$ its value from Eq. 8 results in the differential equation

$$\frac{d^4\phi}{dx^4} - \frac{G\,K + 2\,M\,e}{E\,(I_1\,c^2{}_1 + I_2\,c^2{}_2)}\frac{d^2\phi}{dx^2} - \frac{M^2}{E^2\,I\,(I_1\,c^2{}_1 + I_2\,c^2{}_2)}\phi = 0 \ldots\ldots (68)$$

The solution of this fourth-order linear differential equation, with constants determined from the boundary conditions ($\phi = 0$, $\frac{d^2\phi}{dx^2} = 0$ when $x = 0$ and $x = L$), results in the trigonometric equation

$$\sin L\sqrt{-\frac{G\,K+2\,M\,e}{2\,E\,(I_1 c^2{}_1+I_2 c^2{}_2)} + \sqrt{\frac{(G\,K+2\,M\,e)^2}{4\,E^2\,(I_1 c^2{}_1+I_2 c^2{}_2)^2} + \frac{M^2}{E^2\,I\,(I_1 c^2{}_1+I_2 c^2{}_2)}}} = 0 \ldots (69)$$

The smallest non-trivial root of Eq. 69, solved for the bending moment M, gives, for the critical bending moment,

$$M_{cr} = \frac{\pi^2\,E\,I}{L^2}\left(\sqrt{\frac{L^2\,G\,K}{\pi^2\,E\,I} + \frac{I_1\,I_2}{I^2}\,h^2 + e^2} + e\right) \ldots\ldots\ldots (70)$$

Expressed in the form of Eq. 14*a*, Eq. 70 would read

$$M_{cr} = E\,h\,\frac{\pi^2}{2\,L^2}\left(I\,\frac{2\,e}{h} + \sqrt{4\,I_1\,I_2 + I^2\,\frac{4\,e^2}{h^2} + 4\,C\,I\,\frac{L^2}{\pi^2\,h^2}}\right)\ldots\ldots(71)$$

For the case of a symmetrical I-beam ($I_1 = I_2 = I_0$, and $e = 0$), Eq. 71 reduces to Eq. 14*b*.

The difference between Eqs. 71 and 14*a* is largely in the inclusion of the term e in the former. In the author's solution, no account is taken of the location of the centroid of the cross section. The reason for this discrepancy between the two solutions apparently lies in the method used by the author for evaluating the work of the external forces (Eqs. 10, 11, and 13). Eq. 10 is correct only if the end sections, which rotate through an angle β, remain plane. Under the combined lateral bending and twisting, the end sections do not remain plane. That this is so can be seen from Eq. 11. The change in chord length of a longitudinal fiber varies as the square of the lateral deflection of that fiber at the middle of the span. Since this deflection varies linearly with the depth of the beam, the change in chord length cannot vary linearly with the depth.

To determine the work of the external moment, it is probably necessary to sum up the work done by the forces acting on each unit differential area of the cross section. This would involve the stress distribution on the section and consequently the location of the centroid.

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DISCUSSIONS

PIPE-LINE FLOW OF SOLIDS IN SUSPENSION
A SYMPOSIUM

Discussion

By H. E. Babbitt, M. Am. Soc. C. E., and D. H. Caldwell, Esq.

H. E. Babbitt,[10] M. Am. Soc. C. E., and D. H. Caldwell,[11] Esq.[11a]—The contribution of Professor Wilson to the knowledge of the flow of solids in suspension is of value in providing information applicable to the turbulent flow of settleable solids suspended in water flowing in a circular pipe. The analysis is applicable to the condition of turbulent flow in which the relation between the settling velocity (V_s) of the suspended particles and the transporting velocity (V) of flow is appreciable in terms of the head loss $\left(\frac{H}{L}\right)$. Professor Wilson clearly states that his analysis is limited to the consideration of the transportation of noncolloidal, inert solids for which the average velocity of settling relative to the transporting liquid is known. It is apparent that the analysis is limited to mixtures in which the concentration of suspended solids is insufficient to allow mutual contact between the particles in such a manner that stress may be transmitted through the mixture. It is to be noted that the analysis does not supply information concerning the velocity required to maintain the particles in suspension; nor, from the information given, would it be possible to determine the head losses due to friction for the flow of any given mixture of solids and water.

Information given by the writers[12] in 1939 and 1940 supplements that given by Professor Wilson. Laminar flow and turbulent flow of sludges in circular pipes were discussed, and a method was given for computing head losses due to the flow, in a circular pipe, of mixtures of solids and dispersion fluids through which stress may be transmitted. The writers' analysis covers the transporta-

Note.—This Symposium was published in October, 1941, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1941, by Arthur L. Collins, Assoc. M. Am. Soc. C. E.; and December, 1941, by H. A. Einstein, Assoc. M. Am. Soc. C. E.

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[11a] Received by the Secretary January 14, 1942.

[12] "Laminar Flow of Sludges in Pipes, with Special Reference to Sewage Sludge," by H. E. Babbitt and D. H. Caldwell, *Bulletin No. 319*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1939; also "Turbulent Flow of Sludges in Pipes," by the same authors, *Bulletin No. 323*, *ibid.*, 1940.

tion of solids through circular conduits under conditions in which the settling velocity, V_s, of the suspended particles is negligible with respect to the transporting velocity.

If there is sufficient concentration of suspended particles to permit the transfer of stress, the mixture is said to have a yield value. This is defined as the shearing stress (τ_y) of the mixture, at which flow is impending. High concentrations of suspended particles commonly have a yield value. The existence of a yield value causes the mixture flowing in a pipe to exhibit a pseudoviscosity, higher than the viscosity of the liquid in which the particles are suspended, but decreasing as the velocity in laminar flow increases. A result is that the mixture moves in laminar flow in much the same manner as a very viscous liquid except that, in the case of the mixture, there is a moving plug of material in the center of the pipe and within which there is no movement between adjacent particles. The writers have shown that the velocity, V_c, at which so-called "plug flow," plastic flow, or laminar flow breaks into turbulent flow, is given by the expression

$$V_c = \frac{32\,\eta + 32\sqrt{\eta^2 + \dfrac{f\,\tau_y\,\rho\,D^2\,g}{0.96}}}{f\,\rho\,D} \qquad (26)$$

in which η = coefficient of rigidity of mixture, in pounds per foot-second; f = friction factor in the expression

$$H = f\,\frac{L}{D}\,\frac{V^2}{2\,g} \qquad (27)$$

τ_y = shearing stress at the yield point of a plastic material, called yield value, in pounds per square foot; ρ = density of flowing substance, in pounds per cubic foot; and H = difference in static head between two points in a pipe, measured or expressed in feet of the flowing mixture. Hence, when V is less than V_c, Professor Wilson's analysis cannot be expected to apply.

An analysis of the experimental data reported by the writers, particularly for a Tennessee ball clay slurry, shows that the values of V_c lie between 2.0 and 15 ft per sec. The rate of settling of this material was of the order of 2 to 3 in. per day. Hence, at the velocity of 2 ft per sec (the lowest at which turbulence occurred in the writers' tests), the influence of the term $\frac{K\,p\,V_s}{V}$, shown in Eq. 19, on the friction loss, was negligible when compared with the term $\frac{f\,V^2}{2\,g\,D}$.

When the velocity of flow is greater than V_c, the friction loss due to the flow of a slurry, through a circular pipe, will be the same as the friction loss for the flow, through a circular pipe, of a liquid with the same viscosity as the dispersion medium of the mixture. Hence, in the case of suspensions of solid particles in water, the common hydraulic formulas can be used. This implies that for such a suspension the conditions of flow in a circular pipe can be expressed as

$$\frac{H}{L} = \frac{f\,V^2}{2\,g\,D} \qquad (28)$$

This agrees with Professor Wilson's conclusion. It should be emphasized again that Eq. 28 is applicable only to turbulent flow at a velocity high enough to maintain the particles in suspension.

Importance should be attached to the limiting concentration of suspended matter above which a yield value appears. It is only by differentiating the regions of solids concentration above and below the existence of a yield point that confusion may be avoided. When the concentration of suspended particles is sufficiently low to show little or no yield value in the mixture, when the term $\frac{K\,p\,V_s}{V}$ (Eq. 19) becomes appreciable with respect to the term $\frac{f\,V^2}{2\,g\,D}$, when the flow remains turbulent, and when particles remain in suspension, Professor Wilson's analysis will prove of most value.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ALLOCATION OF THE TENNESSEE VALLEY AUTHORITY PROJECTS

Discussion

By E. L. Chandler, M. Am. Soc. C. E.

E. L. Chandler,[4] M. Am. Soc. C. E.[4a]—Proper allocation of the costs of multiple-purpose projects is a complicated problem, and the author has made a useful contribution to available data on the subject.

The alternative-justifiable-expenditure theory of allocation (see heading "Theories of Joint-Cost Allocations: Alternate Justifiable Expenditure) results in a solution that is applicable to most allocation problems. It appears to warrant more wholehearted approval than that accorded to it by the TVA Financial Policy Committee. In the introduction to its "Notes on Allocation," a supplement to the TVA formal report on cost allocations (H. Doc. No. 709, 75th Cong.), the Committee states that it "found most merit in the 'alternative-justifiable-expenditure' theory" and, in the last sentence of the paragraph on "Conclusion as to Allocation Theories," states that the "allocations finally recommended are admittedly based on an exercise of judgment after considering all facts."

A summary of allocations to date is given in Table 2. By comparing the percentages shown in items Nos. 18, 19, and 20 with corresponding values in items Nos. 31, 32, and 33, it will be noted that the exercise of judgment resulted in a change of 1.6% of the allocation to power in the three-plant system, as compared with the value determined by calculations based on the justifiable-expenditure formula. Changes in the allocations to navigation and flood control were 0.6% and 1.0%, respectively. The changes happen to have decreased step by step until, for the seven-plant system, there is an increase of only 0.7% of the allocation to power. It is scarcely reasonable to suppose that the accuracy of any allocation can be guaranteed within closer limits than these. Perhaps it is not exceeding the bounds of caution to say that the justifiable-

Note.—This paper by Theodore B. Parker, M. Am. Soc. C. E., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by Malcolm Elliott, M. Am. Soc. C. E.

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[4a] Received by the Secretary January 23, 1942.

expenditure method is the adopted method. There is merit in standardization if conditions warrant it.

Added confidence in the merit of this method is found in the fact that "it held its own," from the early days of allocation study, against all of the proposed methods, some of which were ardently supported by their sponsors. It was first used under the title of "benefit theory," and tentative allocations for the first four projects were prepared in 1936 and 1937. The term "benefit theory" was distasteful to some who were concerned with development of the allocations, chiefly because they did not find it possible to arrive at measures of benefits that seemed satisfactory to them. Later, with some changes in terminology and many improvements in details of the procedure used in deriving the estimated justifiable expenditures, but with no change in the application of the formula, the method emerged under the name "alternative-justifiable-expenditure" theory. The writer considers this to be very satisfactory nomenclature.

The maximum total justifiable expenditure for any one of the objectives served by a multiple-purpose project would be measured by the total benefit (with respect to the purpose under consideration) that is derived from the development of the project. Assuming that the value of benefits for any given objective exceeds the cost of the least expensive single-purpose project that might be devised for accomplishing equivalent results, the expenditure necessary for such a single-purpose project may be considered as a lower limit of justifiable expenditure. If the expenditure necessary for such a single-purpose project would not be justified by results to be accomplished, then a lower value for justifiable expenditure should be substituted. It should be some measure of the benefits to result from the expenditure.

The most suitable values to be used as "justifiable expenditures" in any given allocation problem may be determined in any one of several ways, as circumstances dictate. Perhaps a suitable measure of total benefits accruing to each of the purposes may be feasible. It may be found that values based on use of facilities afford a suitable basis, etc. In any event, the values used for all of the objectives of the problem at hand should be maintained on as nearly an equivalent basis as possible. In the TVA allocation, estimates for the cost of alternative single-purpose projects, deemed capable of providing services equivalent to that furnished by the multiple-purpose project, have been used for the purpose.

It is obvious that the accuracy of final allocations under this method depends largely on the soundness of estimates for the alternative justifiable expenditures. Assuming the adequacy of those estimates, the steps taken in developing the TVA allocation give results within what are, probably, suitable limits of accuracy, although they are not absolutely correct. During the allocation studies, methods of derivation were developed that lead to mathematically accurate results for all steps after original estimates have been established.

Method of Determining Directly Segregable Portions of Multiple-Purpose Project Costs.—For any multiple-purpose project, probably certain parts of the expenditures will have been made solely in the interests of specific objectives such as flood control, navigation, or power generation. It also will be true that certain parts of the structures serve more than one purpose. The first step in

the problem of allocation is a determination of the directly segregable portions of the cost. A method for determining those segregable amounts has been developed geometrically and arithmetically as indicated in Fig. 2. This

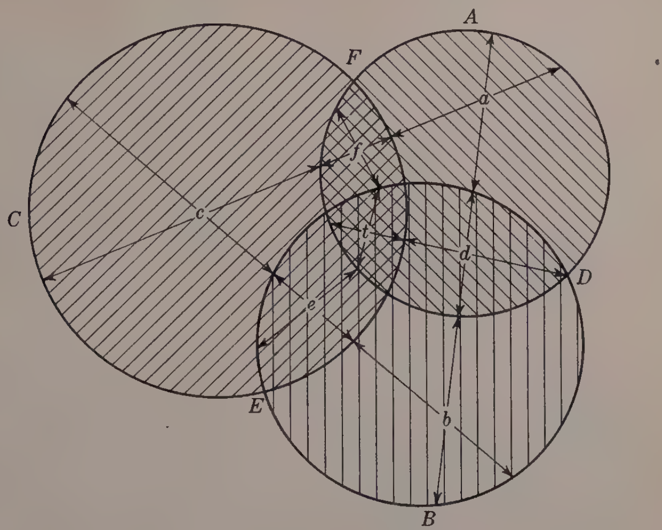

FIG. 2.—SEGREGATION OF DIRECTLY ALLOCABLE PARTS OF THE COST OF A MULTIPLE-PURPOSE PROJECT

diagram and the following computations are intended to establish principles that may be applied to the solution of any given allocation problem:

Area (see Fig. 2)	Description	
	Complete circle; total project cost if built for—	
A	Flood control only	
B	Navigation only	
C	Power only	
	Combined total project cost if built for—	
D	Flood control and navigation (overlapping combination of areas A and B)	
E	Navigation and power (overlapping combination of areas B and C)	
F	Power and flood control (overlapping combination of areas C and A)	
T	Flood control, navigation, and power (overlapping combination of areas A, B, and C; not indicated in Fig. 2)	
	Part of circle not overlapped in combining the three circles is the cost directly allocated to—	
a	Flood control,	$a = T - E$ . . . . (1a)
b	Navigation,	$b = T - F$ . . . . (1b)
c	Power,	$c = T - D$ . . . . (1c)

Single overlapping area common to two circles is the cost to be prorated between—

d Flood control and navigation

$$d = E + F - C - T \quad \text{(2a)}$$

e Navigation and power,

$$e = D + F - A - T \quad \text{(2b)}$$

f Power and flood control,

$$f = D + E - B - T \quad \text{(2c)}$$

Double overlapping area common to circles A, B, and C is the cost to be prorated between—

t Flood control, navigation, and power,

$$t = A + B + C + T - D - E - F \quad \text{(3)}$$

This device has been termed the "three-circle method," although the circles have no special significance. They are used only because they furnish a convenient means of demonstration.

It is considered that three objectives are involved—flood control, navigation, and power development. The first step is a determination of justifiable expenditure for a project that would provide flood control equally satisfactory, in every way, to that furnished by the multiple-purpose project (area A, Fig. 2) but that would provide neither navigation nor power facilities. In like manner, the justifiable expenditure must be determined for a structure that would provide only for navigation equal to that furnished by the multiple-purpose project (area B, Fig. 2), and also for a single-purpose project (area C, Fig. 2) that would furnish the equivalent of power facilities included in the multiple-purpose development.

It is assumed that, if a dual-purpose project were built to provide for flood control and navigation, the resulting expenditure would be less than the combined sum represented by the total of areas A and B. Therefore, to represent such a dual-purpose project graphically, the two circles are shown overlapping in the diagram, and the combination is designated as area D, which is less than the total area A plus the total area B by the amount of overlapping (in this instance, indicated by the area d plus t). In the same way, if a dual-purpose project were to be built to serve the purposes of navigation and power, with no provision for flood control, the resulting expenditure is represented on the diagram by that part labeled E, which includes overlapping areas B and C. The net resulting sum in this case is less than the combined total of areas B and C by the amount of overlapping between the two areas, which is graphically represented by area e plus area t, Fig. 2. In the same way, a dual-purpose project built for purposes of flood control and power only is represented by the overlapping of area A and area C, the area being less than the combined total of A plus C by the amount of overlapping indicated by area f plus t. The three purposes are served at a cost materially less than would be the case if each of the

three were to be gained by a single structure or if any combination of dual-purpose and single-purpose projects were developed.

Eqs. 1 to 3 will give the proper part of the total cost to be allocated directly to each of the three objectives; the part that is common solely to each combination of two objectives; and that part common to all three of the objectives. These values are fundamental, and allocation of the cost of the tri-purpose project is based on them.

In the case of a project involving more than three objectives, the method outlined would still be applicable. A fourth circle would be introduced and a larger number of simultaneous equations would result, but the principles involved would remain the same.

The discussion thus far has been purely theoretical. Segregation of the cost of a hypothetical project in accordance with the aforementioned theory is indicated arithmetically and diagrammatically in Fig. 3. The incremental

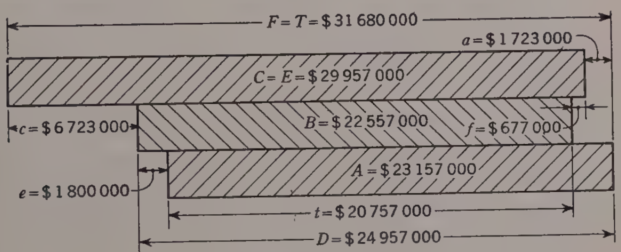

FIG. 3

costs are those expenditures made solely in the interest of single purposes. The first step in such a segregation is a determination of the justifiable expenditures for single-purpose projects that would have to be built to satisfy each one of the three objectives, and also for those projects that would serve each of the three dual combinations to an extent equal to the service rendered by the actual tri-purpose project. The following tabulation indicates the assumed cost of the project that would have resulted in each case:

Area (see Fig. 2)	Objective	Cost
A	Flood control only	$23,157,000
B	Navigation only	22,557,000
C	Power only	29,957,000
D	Flood control and navigation	24,957,000
E	Navigation and power	29,957,000
F	Flood control and power	31,680,000
T	Total	31,680,000

For the sake of clarity, in Fig. 2 the areas are shown overlapping by only small segments. As a matter of fact, in the case of most projects the cost common to two or more objectives is likely to be a large proportion of the total cost, and such a condition has been assumed for the hypothetical example. Then, it is difficult to indicate the true conditions to scale by the use of circles, and a different type of diagram has been devised in Fig. 3.

A study of this diagram will show that the cost of a project that would give flood-protection facilities equivalent to those assumed would be \$23,157,000. Such a project would not be built in the interests of navigation or power development. This is indicated in Fig. 3 by the lower rectangle, length A. A project that would serve purposes of navigation only, without regard to flood control or to power, would cost \$22,557,000 (length B, Fig. 3). The cost of a project built for purposes of power generation only would be \$29,957,000 (length $C = E$, Fig. 3). The three rectangles have been arranged in a horizontal position such that the various other elements of cost desired are shown diagrammatically. For instance, if a dual-purpose project were built that would furnish flood-control and navigation facilities equivalent to those furnished by the tri-purpose project, but which would provide nothing specifically for power development, the cost would be \$24,957,000. This is represented by length D.

The hypothetical tri-purpose project is such that, if a single-purpose project were built for the primary purpose of providing an equivalent of the existing power generation facilities, it would be sufficient also to provide the existing navigation facilities without additional cost. This is found to be true in the case of some of the TVA projects on the upper tributaries of the Tennessee River, where the value to navigation consists of releasing water during periods of normally low flow in the channel downstream. This means, in the case of the circles in Fig. 2, that area B, representing navigation, would fall entirely within area C, representing power. The cost of a dual-purpose project for navigation and power becomes the same as that for a single-purpose project for power. Thus, in Fig. 3 the top rectangle is marked both C and E. It also has been assumed that a dual-purpose project that would provide flood-control facilities and power-generation facilities equivalent to those now existing would be the same as the existing structure. Therefore, the combined dual cost represented by length F (Fig. 3) would be the same as the total cost of the existing project, T (= \$31,680,000). The assumptions have been made with the purpose of setting up a rather complicated example.

Costs represented by lower-case letters and the overlapping of circles in the theoretical discussion have been determined as follows:

The cost directly allocable to flood control, represented by length a, is \$1,723,000. In Fig. 3 it is represented by that part of the lower rectangle which extends beyond the right-hand limit of the upper rectangles. A project that would provide suitable power-generation facilities would automatically provide the necessary navigation facilities. Therefore, there is no part of the cost of the project to be allocated directly to navigation and, consequently, the value of b becomes zero. That part of the cost properly allocable solely to power generation, represented by length c, is \$6,723,000. This is the part of the top rectangle which extends to the left beyond the limit of the lower rectangles. There is no value indicated for length d which, in the theoretical discussion, is that part of the cost common to flood control and navigation only. Considering only a dual-purpose project for purposes of flood control and navigation, the common element length d becomes \$20,757,000. However, a dual-purpose project involving navigation and power would cost the same as a single-purpose project for power only. Thus, in Fig. 3, rectangle B, repre-

senting navigation, is completely overlapped by rectangle C, representing power, and it is evident that any part of the cost common to lengths A and B also must be common to length C. Therefore, the sum of $20,757,000, which is common to lengths A and B, actually becomes length t in this study (it is common to all three of the objectives), and d becomes zero. This means that no part of the cost is common to costs A and B which is not also common to cost C. That part of the cost common to navigation and power only (length e, Fig. 3) is $1,800,000. That part of the cost common to flood control and power only (length f, Fig. 3) is $677,000. The sum of $20,757,000, common to all three of the objectives, is shown as length t in Fig. 3.

The foregoing procedure appears to be mathematically correct. The fact that some of the values may prove to be zero has no bearing at all on the soundness of the method. The matter of resolving the total cost of a multiple-purpose project into the incremental costs directly allocable to the several objectives and into costs that are common is relatively simple if the foregoing procedure is followed. Dependability of the results obtained depends wholly on the accuracy with which the costs represented by the several capital letters are determined.

To segregate the costs for a tri-purpose project, it will be noted that estimates of cost for seven different projects may be required, although, as in the foregoing hypothetical case, the number may be reduced if two functions are served by a project built for any one of the objectives. In such a case a single-purpose project automatically assumes the character of a dual-purpose project.

During discussions of the allocation procedures, the estimating required in order to apply the method seemed to be a formidable objection to its use in the minds of some non-engineering members of the TVA staff. It does not appear too serious from an engineering viewpoint. Granting that any estimate is likely to be somewhat in error, carefully prepared estimates, formulated by experienced engineers, should fall within suitable limits of accuracy to satisfy the needs at hand. This is especially true when consideration is given to the relatively minor effect on ultimate allocations that may result from substantial variations in original estimates. This has been demonstrated by Mr. Parker in Table 3. The writer investigated these relations during the course of allocation studies and found that, on a basis substantially the same as the foregoing, with variations as large as 20% in the estimates of justifiable expenditures, the greatest change in allocation was approximately 4%, and the average was somewhat less than 2%. Careful estimates should not be more than 20% out of balance, and there seems little likelihood that allocations can be determined with any certainty of accuracy within 2%.

Method of Allocating Common Costs Under Alternative-Justifiable-Expenditure Theory.—The non-segregable costs common to more than one objective are determined under the "three-circle method" of segregation. In the cases studied, only tri-purpose projects are considered, although the same method of segregation and allocation may be applied to any multiple-purpose project.

In the "three-circle method" of segregation an "incremental cost" was determined for each of the three purposes, that being the expenditure incurred in each case over and above what would be expended if a dual-purpose project

were built and the purpose under consideration were non-existent in the project. The costs common to each possible combination of two purposes were established on a basis of hypothetical dual-purpose projects, and the cost common to all three purposes was established.

After determining the "justifiable expenditure" for each objective, the segregated "incremental cost" is deducted, and the result becomes a sum that may be termed "justifiable additional investment." Having already charged to the objectives those parts of the multiple-purpose project costs that are directly segregable, it is logical to allocate the remaining common costs on the basis of "justifiable additional investments."

As described under the three-circle method of segregation, there are four items of common cost to be allocated. For exact allocation, each common cost should not be distributed directly on the basis of the "justifiable additional investments" with which it is concerned. Rather, each should be distributed in proportion to the remainders that would be left after the other three items of common cost have been allocated, and such allocations have been deducted from the corresponding "justifiable additional investments." The resulting amounts may be termed "remaining justifiable additional investments." The development of mathematical equations by which common costs may be directly allocated on this basis follows.

The following symbols are adopted to represent the quantities involved:

Symbol	Description
	Justifiable additional investment for—
M	Flood control
N	Navigation
O	Power
	Cost common to (see Eqs. 2 and 3)—
d	Flood control and navigation only
e	Navigation and power only
f	Power and flood control only
t	Flood control, navigation, and power
	Proportion of d allocated to—
d_m	Flood control
d_n	Navigation
	Proportion of e allocated to—
e_n	Navigation
e_o	Power
	Proportion of f allocated to—
f_m	Flood control
f_o	Power
	Proportion of t allocated to—
t_m	Flood control
t_n	Navigation
t_o	Power

Allocation of t.—The steps in the allocation of cost t are shown in Table 4(a). The total common cost t is to the part of t to be allocated to flood control

(t_m) as the total "remaining justifiable additional investment" is to the remaining justifiable additional investment for the part allocated to flood control.

TABLE 4.—ALLOCATION FORMULAS; JUSTIFIABLE ADDITIONAL INVESTMENT

Justifiable additional investment	(a) ALLOCATION OF t		(b) ALLOCATION OF d		(c) ALLOCATION OF e		(d) ALLOCATION OF f	
	Previously allocated[a]	Break-down	Previously allocated[a]	Break-down	Previously allocated[a]	Break-down	Previously allocated[a]	Break-down
M	$d_m + f_m$	t_m	$f_m + t_m$	d_m	....	....	$d_m + t_m$	f_m
N	$d_n + e_n$	t_n	$e_n + t_n$	d_n	$d_n + t_n$	e_n	....	....
O	$e_o + f_o$	t_o	....	....	$f_o + t_o$	e_o	$e_o + t_o$	f_o
$M+N+O$	$d+e+f$	t	$e_n+f_m+t_m+t_n$	d	$d_n+f_o+t_n+t_o$	e	$d_m+e_o+t_m+t_o$	f

[a] Items considered as if previously allocated.

Expressing this proportion algebraically and transposing t to solve for t_m:

$$t_m = t \frac{M - (d_m + f_m)}{(M + N + O) - (d + e + f)} \qquad (4a)$$

Similarly, the total common cost t is to the part of t allocated to navigation (t_n) as the total remaining justifiable additional investment is to the remaining justifiable additional investment for the part allocated to navigation; thus:

$$t_n = t \frac{N - (d_n + e_n)}{(M + N + O) - (d + e + f)} \qquad (4b)$$

Finally, the total common cost t is to the part of t allocated to power (t_o) as the total remaining justifiable additional investment is to the remaining justifiable additional investment for the part allocated to power:

$$t_o = t \frac{O - (e_o + f_o)}{(M + N + O) - (d + e + f)} \qquad (4c)$$

Allocation of d.—Since it is common to M and N only (see Table 4(b)), d is broken down into d_m and d_n. This eliminates line O, and the comparable equations are—

For flood control:

$$d_m = d \frac{M - (f_m + t_m)}{(M + N) - (e_n + f_m + t_m + t_n)} \qquad (5a)$$

and, for navigation:

$$d_n = d \frac{N - (e_n + t_n)}{(M + N) - (e_n + f_m + t_m + t_n)} \qquad (5b)$$

Allocation of e.—In this case, e is common to N and O only (see Table 4(c)) and therefore is broken down into e_n and e_o. Line M is thus eliminated and—

For navigation:

$$e_n = e \frac{N - (d_n + t_n)}{(N + O) - (d_n + f_o + t_n + t_o)} \qquad (6a)$$

and, for power:

$$e_o = e \frac{O - (f_o + t_o)}{(N + O) - (d_n + f_o + t_n + t_o)} \quad \text{(6b)}$$

Allocation of f.—Being common to M and O only (see Table 4(d)), f is divided into f_m and f_o. This eliminates N, the justifiable additional investment for navigation. The remaining formulas are—

For flood control:

$$f_m = f \frac{M - (d_m + t_m)}{(M + O) - (d_m + e_o + t_m + t_o)} \quad \text{(7a)}$$

and, for power:

$$f_o = f \frac{O - (e_o + t_o)}{(M + O) - (d_m + e_o + t_m + t_o)} \quad \text{(7b)}$$

Major Equations.—First eliminate Eqs. 4 by substitution. For simplicity these may be written:

$$t_m = \frac{t}{K} (M - d_m - f_m) \quad \text{(8a)}$$

$$t_n = \frac{t}{K} (N - d_n - e_n) \quad \text{(8b)}$$

and

$$t_o = \frac{t}{K} (O - e_o - f_o) \quad \text{(8c)}$$

in which

$$K = (M + N + O) - (d + e + f) \quad \text{(9)}$$

In Eq. 5a (for example), substitute the values of t_m and t_n defined by Eqs. 8, and simplify the resulting form algebraically to

$$d_m = d \left[\frac{L (M - f_m) + t\, d_m}{L (M + N - e_n - f_m) + t\, d} \right] \quad \text{(10)}$$

in which $L = K - t$. Eq. 10 may be simplified further to the general formula:

$$d_m (M + N - e_n - f_m) = d (M - f_m) \quad \text{(11)}$$

Similarly, Eqs. 8 substituted in Eq. 6a yield (simplified algebraically)—

$$e_n = e \left[\frac{L (N - d_n) + t\, e_n}{L (N + O - d_n - f_o) + t\, e} \right] \quad \text{(12)}$$

which may be simplified to the general formula

$$e_n (N + O - d_n - f_o) = e (N - d_n) \quad \text{(13)}$$

In Eq. 7b, substitute t_o and t_m from Eqs. 4 to yield

$$f_o = f \left[\frac{L (O - e_o) + t\, f_o}{L (M + O - d_m - e_o) + t\, f} \right] \quad \text{(14)}$$

or

$$f_o (M + O - d_m - e_o) = f (O - e_o) \quad \text{(15)}$$

Eqs. 11, 13, and 15 can be written in the following form:

$$(M + N - f + f_o - e_n)\, d_m = d\,(M - f + f_o) \qquad \text{(16a)}$$

$$(N + O - d + d_m - f_o)\, e_n = e\,(N - d + d_m) \qquad \text{(16b)}$$

and

$$(O + M - e + e_n - d_m)\, f_o = f\,(O - e + e_n) \qquad \text{(16c)}$$

Solving Eq. 16c for f_o and substituting this value in Eqs. 16a and 16b, successively:

$$d_m\,[(O + M - e + e_n - d_m)\,(M + N - e_n - f) + f\,(O - e + e_n)] = d\,[(O + M - e + e_n - d_m)\,(M - f) + f\,(O - e + e_n)] \qquad \text{(17a)}$$

and

$$e_n\,[(O + M - e + e_n - d_m)\,(N + O - d + d_m) - f\,(O - e + e_n)] = e\,[(N - d + d_m)\,(O + M - e + e_n - d_m)] \qquad \text{(17b)}$$

The foregoing derivations have been given in considerably more detail than is necessary to obtain the results but the full procedure is given for the sake of clarity. Beyond Eqs. 17, further modification is inadvisable. By substituting known quantities in these formulas, solutions of d_m and e_n may be obtained. It is then a simple matter to derive allocations of the other common costs by substituting those values in preceding equations. Frequently, some one or more of the hypothetical dual-purpose projects contains no cost that is common only to the two objectives used in combination (that is, d, e, or $f = 0$). Under such circumstances, Eqs. 17 become much simplified. If desired, answers may be obtained by a graphic solution of the equations. In some instances the graphic method will be found to be preferable.

"Cut-and-Try" Solution.—The foregoing equations give results that are mathematically accurate. However, in most instances it is possible to arrive at results that are within satisfactory limits of accuracy by using a cut-and-try method of solution, which involves considerably less work without departing from the fundamental principles of the equations.

As a first step in the cut-and-try method, d, e, and f are broken down in direct proportion to the "justifiable additional investments," M, N, and O. This gives a set of tentative allocations for d, e, and f. Based on these values, a set of tentative "remaining justifiable additional investments" is obtained. These form a basis for a tentative allocation of t. Then the latter allocated values (t_m, t_n, and t_o) exchange places with the tentative breakdown of one of the first three common costs (say f), and a second set of "remaining justifiable additional investments" is obtained. Based on these, f is reallocated.

Following a similar procedure, the latest allocated values (f_o and f_m) now exchange places with the tentative values for e, and a new allocation for e is determined. Extending the process another step, d is reallocated. This completes a cycle that furnishes a new breakdown for each of the common costs. By the nature of the process these values are closer to a true allocation of the common costs than those first used.

It will be noted that these tentative allocations have been based on "remaining justifiable additional investments," but that the latter are not those

which would result from deducting the true allocations from the original "justifiable additional investments."

TABLE 5.—CUT-AND-TRY SOLUTION
(Units Are Dollars)

JUSTIFIABLE ADDITIONAL INVESTMENT		APPROXIMATE BREAKDOWN OF MULTIPLE-PURPOSE COSTS				Remaining justifiable investment	ALLOCATION OF MULTIPLE-PURPOSE COMMON COST			
Symbol	Cost (1)	*d* (2)	*e* (3)	*f* (4)	*t* (5)	(6)	*t* (7)	*f* (8)	*e* (9)	*d* (10)
					(*a*) FIRST CYCLE					
Allocation of *t*:										
M	500	269	...	133	....	98	56	....	....	....
N	800	431	222	...	....	147	84	....	....	....
O	1,000	...	278	267	....	455	260	....	....	....
M+*N*+*O*	2,300	700	500	400	....	700	400	....	....	....
Allocation of *f*:										
M	500	269	...	....	56	175	....	109.89	....	....
N	800	431	222	....	84	63	....	....	....	....
O	1,000	...	278	....	260	462	....	290.11	....	....
M+*N*+*O*	2,300	700	500	....	400[a]	700	....	400	....	....
Allocation of *e*:										
M	500	269	....	109.89	56	65.11	....	....	....	....
N	800	431	....	....	84	285.00	....	....	193.91	....
O	1,000	...	....	290.11	260	449.89	....	....	306.09	....
M+*N*+*O*	2,300	700	....	400[b]	400[a]	800	....	....	500	....
Allocation of *d*:										
M	500	....	....	109.89	56	334.11	....	....	....	273.16
N	800	....	193.91	....	84	522.09	....	....	....	426.84
O	1,000	....	306.09	290.11	260	143.80	....	....	....	....
M+*N*+*O*	2,300	....	500[c]	400[b]	400[a]	1,000	....	....	....	700
					(*b*) SECOND CYCLE					
Allocation of *t*:										
M	500	273.16	....	109.89	....	116.95	66.83	....	....	....
N	800	426.84	193.91	....	....	179.25	102.43	....	....	....
O	1,000	....	306.09	290.11	....	403.80	230.74	....	....	....
M+*N*+*O*	2,300	700	500	400	....	700	400	....	....	....
Allocation of *f*:										
M	500	273.16	....	....	66.83	160.01	....	102.70	....	....
N	800	426.84	193.91	....	102.43	76.82	....	....	....	....
O	1,000	....	306.09	....	230.74	463.17	....	297.30	....	....
M+*N*+*O*	2,300	700	500	....	400[a]	700	....	400	....	....
Allocation of *e*:										
M	500	273.16	....	102.70	66.83	57.31	....	....	....	....
N	800	426.84	....	....	102.43	270.73	....	....	182.26	....
O	1,000	....	....	297.30	230.74	471.96	....	....	317.74	....
M+*N*+*O*	2,300	700	....	400[b]	400[a]	800	....	....	500	
Allocation of *d*:										
M	500	....	....	102.70	66.83	330.47	....	....	....	273.51
N	800	....	182.26	....	102.43	515.31	....	....	....	426.49
O	1,000	....	317.74	297.30	230.74	154.22	....	....	....	....
M+*N*+*O*	2,300	....	500[c]	400[b]	400[a]	1,000	....	....	....	700

[a] *t*(=\$400) in Col. 5 taken from preceding determination of *t* in Col. 7. [b] *f*(=\$400) in Col. 4 taken from preceding determination of *f* in Col. 8. [c] *e*(=\$500) in Col. 3, taken from preceding determination of *e*(=\$500) in Col. 9.

TABLE 5.—(*Continued*)

Justifiable Additional Investment		Approximate Breakdown of Multiple-Purpose Costs				Remaining justifiable investment	Allocation of Multiple-Purpose Common Cost			
Symbol	Cost (1)	*d* (2)	*e* (3)	*f* (4)	*t* (5)	(6)	*t* (7)	*f* (8)	*e* (9)	*d* (10)
(*c*) Ninth Cycle (Following Successive Intermediate Cycles)										
Allocation of *t*:										
M	500	272.28	....	101.02	....	126.70	72.40	....	....	....
N	800	427.72	173.48	....	....	198.80	113.60	....	....	....
O	1,000	....	326.52	298.98	....	374.50	214.00	....	....	....
$M+N+O$	2,300	700	500	400	....	700	400	....	....	....
Allocation of *f*:										
M	500	272.28	....	....	72.40	155.32	....	101.05	....	....
N	800	427.72	173.48	....	113.60	85.20	....	....	....	....
O	1,000	....	326.52	....	214.00	459.48	....	298.95	....	....
$M+N+O$	2,300	700	500	....	400[a]	700	....	400	....	....
Allocation of *e*:										
M	500	272 28	....	101.05	72.40	54.27	....	....	....	....
N	800	427.72	....	....	113.60	258.68	....	....	173.44	....
O	1,000	....	....	298.95	214.00	487.05	....	....	326.56	....
$M+N+O$	2,300	700	....	400[b]	400[a]	800	....	....	500	....
Allocation of *d*:										
M	500	....	....	101.05	72.40	326.55	....	....	....	272.28
N	800	....	173.44	....	113.60	512.96	....	....	....	427.72
O	1,000	....	326.56	298.95	214.00	160.49	....	....	....	....
$M+N+O$	2,300	....	500[c]	400[b]	400[a]	1,000	....	....	....	700

[a] $t(=\$400)$ in Col. 5 taken from preceding determination of t in Col. 7. [b] $f(=\$400)$ in Col. 4 taken from preceding determination of f in Col. 8. [c] $e(=\$500)$ in Col. 3, taken from preceding determination of $e(=\$500)$ in Col. 9.

Based on these allocations, other cycles are computed. With each cycle the adjustment in allocations becomes less and less as the true values are approached. The process is continued until the adjustments fall within desired limits of accuracy.

A hypothetical example is presented in Table 5, in which each of the aforementioned steps are shown. Nine cycles were used in order to study the rate at which the adjustment in the allocation values diminished. Computations for cycles intervening between the second and ninth are omitted since they add little to clarifying the procedure. Completion of the fourth cycle brought the allocation into such close agreement with the true values that further work was not justified with the small gain in accuracy. Hence, four cycles are deemed sufficient for a practical application of the method. In practice it is possible to omit the values of t and solve for d, e, and f; but, as in the development of the fundamental equations, all of the steps are included for the sake of clarity.

This cut-and-try method was found to be very satisfactory in the solution of the problem. A check may be made on the calculations at any time, and the work stopped after the degree of accuracy justified has been obtained. The variation of results obtained by each cycle indicates the rate at which the

allocation values are approaching correctness. By studying these reductions it is readily possible to determine when the results have reached a point beyond which the refinement gained makes further calculations unnecessary.

The foregoing demonstration develops a mathematically correct method of allocating the joint or common costs in any multiple-purpose project, although in most instances it may be found that the increase in accuracy which is gained by the refinements of allocating the costs common only to two objectives (d, e, and f in the equations) is not of sufficient consequence to justify the work required. Under such circumstances it should be sufficient to allocate all of the joint costs ($d + e + f + t$) in one operation in proportion to the "justifiable additional expenditures" and to omit the other steps.

Acknowledgment.—The fundamentals of the foregoing mathematical derivations were proposed by Sherman M. Woodward, M. Am. Soc. C. E., chief water control planning engineer, under whose direction the writer acted as chief estimator in the TVA organization.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PROTECTIVE AND REMEDIAL MEASURES FOR SANITARY AND PUBLIC HEALTH ENGINEERING SERVICES

Discussion

BY DONALD M. BAKER, M. AM. SOC. C. E.

DONALD M. BAKER,[4] M. AM. SOC. C. E.[4a]—For those public agencies engaged in Sanitary and Public Health Engineering services, this Report furnishes an excellent manual, but the writer, basing his feelings upon personal experience gained while serving on local defense committees during the year 1940–1941, believes that insufficient stress is given to the subject of Paragraph 5(*a*) of the Report, "the likelihood of being sabotaged or attacked."

Every local agency, public or private, engaged in the operation or maintenance of the aforementioned type of utility or service, or in fact of any type of utility, becomes extremely enthusiastic in placing its utility or service in the best possible condition to meet and cope with even the most remote emergency arising out of war conditions, threatened or actual.

Balance is needed in such a program; otherwise unnecessary expenditures are likely to be made, and the public unduly alarmed, by the activities of an aggressive administrator who feels that his own utility or service is surely the most immediate objective of attack by saboteurs or by enemy action, and that the damage thereto will have the most serious effect of any damage which might occur in the community from such action. As a matter of fact there may be a dozen others far more probable of receiving attention, whose damage would be far more serious.

The British have found by experience that it is economically (and often physically impossible) to provide 100% protection for all life and property in every community, against every conceivable contingency. The probability of damage to, and disruption of, each type of utility and service, their relative exposure to such damage, the ease with which damage can be inflicted, and its

NOTE.—This Progress Report of the Sanitary and Public Health Engineering Division of the National Committee of the Society, on Civilian Protection in War Time, was published in January, 1942, *Proceedings*.

[4] Cons. Engr., Los Angeles, Calif.

[4a] Received by the Secretary February 2, 1942.

effect upon public safety, health, and morale must be considered, and a final program adopted which is based upon these considerations. This requires the same type of investigation and judgment as is exercised in planning flood protection works, where a balance must be struck between magnitude and frequency of floods expected to occur, the cost of protecting against them, and the damage likely to occur with varying degrees of protection.

Each community presents a different problem. No standard plan of protection can be developed. Exposure of utilities and services in each will differ greatly, as will the effects of damage. Each defense organization must be set up on a basis sufficiently widespread to embrace the entire area requiring protection. Provision must be made for the highest degree of coordination, and the interchange of equipment, supplies, material, and personnel. Just as interconnection between power systems serves to reduce installed generating capacity required to supply power demands of a region, so will such type of organization and coordination reduce the equipment, supplies, etc., required to cope with emergencies.

Sabotage can be expected in any locality at any time during the present emergency, and from the following sources: (*a*) "Cranks," (*b*) enemy sympathizers (individuals or small groups) working without direction or any general plan of operations, and (*c*) enemy sympathizers (individuals or groups) working under direction and in accordance with a carefully directed and prepared plan.

Any emergency stimulates "cranks" to action. Their activities are more of a nuisance character; they seldom cause serious results or damage, and can usually be guarded against by routine vigilance. Unorganized and undirected enemy sympathizers are a more serious problem because, although saboteurs of this type are probably few in number, it is not possible to forecast what their course of action will be or where they will strike, their primary objective being to cause trouble or damage, irrespective of the crippling effect of their activities. Counter espionage and vigilant protection of exposed utilities and services are the best remedies against this type of saboteur.

Organized and directed sabotage is only undertaken when its probable result will seriously affect public morale, production of important war material, or operation of military or naval forces. Probable results to be achieved usually are carefully weighed against probable effects. So far, the United States has been reasonably free from sabotage activities, except those of cranks. This is undoubtedly due to excellent protective measures instituted by federal, state, and local public and private agencies. As time goes on, however, it can be expected that serious efforts toward major sabotage activities of this type will be made.

Danger of attack from gunfire exists in communities along the seacoast. Such attack can be expected only as a result of a carefully organized and prepared plan, where the results achieved, in damage to public morale, diversion of protective forces from offensive or defensive action elsewhere, or in damage to production or to military or naval facilities, will more than warrant the cost of such attack to enemy material or personnel. Such attack will come in surprise form and with small force under present conditions, and its current probability is more or less remote. As the Axis nations, with time, find themselves in more desperate circumstances, however, they will be prone to use less caution in balancing cost against results, and probability of this type of attack will increase.

Danger from air attack extends to communities farther inland. At the present time it could only be expected from carrier-based planes, and in any event would be sudden, in relatively small force, and not persistent. The lessons of Pearl Harbor have decreased greatly the probability of such occurrence. Although Axis nations may be developing transoceanic bombers that could successfully negotiate a round-trip voyage and conduct a raid at this end, the probability is that, for the present at least, any ships of this type developed could be used with far greater effect in other places, and little more than token raids could be expected. As in the case of attack by gun fire, with increasing desperation on the part of the Axis powers, such attacks might become a less remote probability.

The first step to be taken in any wartime protection program by local defense agencies should embrace consultation with military and naval authorities to determine the types of attack to be expected, the objects most likely to be attacked, and the relative probability of each type of attack upon each kind of objective. This will allow protective measures to be taken with greatest efficiency and economy, and a balanced program developed. Such contact should be maintained throughout the period of emergency, as with the changing fortunes of war these conditions will undergo constant change.

Corrections: January, 1942, *Proceedings*, page 121, line 5, change "1,000 gal per 24 hr" to "900 gal per hr"; page 129, change the heading of Paragraph 31 to read "31. Refuse Disposal Works"; in the "Bibliography," in Reference 18 under the heading "General," after "Stanton Iron Works," add "Nottingham, England"; page 105, line 4, the third sentence of the report should read: "A list of the Sanitary and Public Health Engineering representatives of these sub-committees appears at the end of the Report"; and, on page 135, line 15, after "64 Local Sections of the Society," insert "The Sanitary and Public Health Engineering representatives in these Local Sections are as follows:

H. H. Hendon, 51 Norman Drive, Birmingham, Ala.
Miss Jane H. Rider, 534 West Latham St., Phoenix, Ariz.
George E. Symons, Buffalo Sewer Authority, Bird Island Laboratory, Buffalo, N. Y.
H. E. Babbitt, 204 Eng. Hall, Univ. of Illinois, Urbana, Ill.
F. H. Waring, State Dept. of Health, Depts. of State Bldg., North Front St., Columbus, Ohio.
John S. Raffety, 1518 Linn St., Cincinnati, Ohio
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Russell Suter, 114 South Lake Ave., Albany, N. Y.
W. L. Picton, 711 American National Bank Bldg., Nashville, Tenn.
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DISCUSSIONS

STABILITY OF GRANULAR MATERIALS

Discussion

BY STANLEY U. BENSCOTER, JUN. AM. SOC. C. E.

STANLEY U. BENSCOTER,[5] JUN. AM. SOC. C. E.[5a]—The stress analysis of a cohesionless embankment, which the authors have presented, is a definite step forward in the knowledge of the behavior of granular materials. Many contributions, both theoretical and experimental, in this field of stress analysis are needed to create a reasonable conception of the state of stress at all points within a granular mass. This field has been neglected too long, most probably because of the supposedly insurmountable mathematical difficulties involved in dealing with a granular mass. If physicists and aeronautical engineers have been able to treat, mathematically, the behavior of groups of particles having variable velocities in thermodynamics and aerodynamics, soil physicists certainly should be able to treat, mathematically, groups of particles having variable masses. It is almost certain that statistical methods and other branches of advanced mathematics will find application eventually in the stress analysis of soils.

The authors have obtained a solution for stresses in an embankment which is possible of existence so far as can be determined from the geometry of a Mohr diagram and the assumption of a failure law representable as a straight line passing through the origin of the Mohr axes. The division of the embankment and foundation into elastic and plastic regions offers an interesting approach to the problem. The reported behavior of the elastic regions is reasonable since they decrease with increasing height of the embankment. The concept of the reserve strength R is valuable and should find much application in the future.

The geometry of a Mohr diagram yields a quadratic algebraic equation (Eq. 11*b*) relating stresses and the internal friction coefficient in a plastic region. The authors have shown that linear expressions for stresses can be found which satisfy this equation and the two equations of equilibrium of an

NOTE.—This paper by R. E. Glover, Esq., and F. E. Cornwell, Esq., was published in November, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1941, by R. G. Hennes, Assoc. M. Am. Soc. C. E.

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[5a] Received by the Secretary February 2, 1942.

element. As the authors state, future investigations must be made to find the possibilities of expressing stresses in a plastic region by higher degree polynomials or transcendental functions. This suggests that several solutions may be found for stresses that are possible of existence; and there will probably be a need for one or more additional laws to govern the solution before the correct one (or the most probable one) can be chosen.

If the stresses are expressed in terms of an Airy function, thus assuring equilibrium of elements, the quadratic algebraic equation becomes a quadratic differential equation. Prof. Theodor von Kármán, M. Am. Soc. C. E., drew attention to this equation as one among many non-linear differential equations with which the engineer is struggling.[6] It seems strange that government engineering agencies interested in the behavior of soil are not financing a program of mathematical research in the behavior of granular masses.

Another manner of dealing with the non-linear equation governing stresses in a granular mass is the use of the principal stress lines as a coordinate system. In this orthogonal curvilinear system the shearing stress vanishes and the non-linear equation becomes linear. The resulting equation is merely a proportionality relation between the principal stresses as given by Eq. 11*a*. A simple example will illustrate: Consider the two dimensional polar coordinate system to form a set of principal stress lines in a granular mass and the body forces to be negligible. The shearing stress $\tau_{r\theta}$ is zero at all points, and the Mohr diagram reveals that

$$\sigma_r = K\,\sigma_\theta \qquad (32)$$

in which K is Rankine's coefficient, either active or passive. The formulas for σ_r and σ_θ, in terms of an Airy function, are

$$\sigma_r = \frac{\partial^2 F}{\partial r^2} \qquad (33a)$$

and

$$\sigma_\theta = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2} \qquad (33b)$$

Assuming symmetry about the origin, try the following simple stress function,

$$F = A\,r^n \qquad (34)$$

Differentiating to obtain σ_r and σ_θ and substituting in Eq. 32:

$$\sigma_r = p\left(\frac{r}{a}\right)^{K-1} \qquad (35a)$$

and

$$\sigma_\theta = \frac{p}{K}\left(\frac{r}{a}\right)^{K-1} \qquad (35b)$$

In Eqs. 35, p is the value of σ_r at $r = a$. These formulas (represented graphically in Fig. 7) indicate the distributions that are possible in a solid or hollow cylinder of dry sand acted upon by a uniform external pressure. The inside

[6] "The Engineer Grapples with Nonlinear Problems," by T. von Kármán, *Bulletin*, Am. Mathematical Soc., Vol. 46, No. 8, August, 1940, pp. 615–683.

pressure of the hollow cylinder may vary between minimum and maximum values. If the inside pressure has an intermediate value, the internal stress distribution is not known. This may be a statistical problem rather than one in elasticity. For the solid cylinder, Curve II must be rejected since it becomes

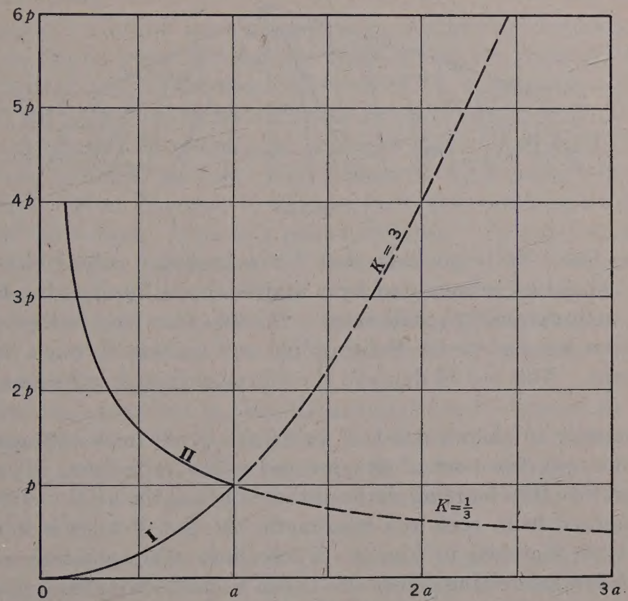

FIG. 7.—VALUES OF σ_r

infinite at the origin. Curve I gives a possible stress distribution; yet it is certainly questionable.

The state of stress at a point in a body, in the two-dimensional case, may be completely defined by giving the stress circle on a Mohr diagram and the directions of principal stresses. The directions of principal stresses at all points within the body may be represented by drawing the principal stress lines. Thus it is apparent that investigators should strive to find laws governing the shape and location of principal stress lines. Today they know only that these lines form two orthogonal curvilinear systems which are continuous even through boundaries between elastic and plastic regions. They are not necessarily representations of harmonic functions, or solutions of La Place's equation, as one might be tempted to hope. In this connection it is interesting to note that, in 1941, M. A. Sadowsky[7] showed that in perfectly plastic behavior (no internal friction) principal stress lines form an equi-areal pattern. In elastic bodies these lines must be parallel and perpendicular to unloaded surfaces. In a granular mass this may not be true.

The principal stress lines for Example 1 (Fig. 1) may readily be drawn as shown in Fig. 8. These lines must be questioned, with a view toward en-

[7] "Equiareal Pattern of Stress Trajectories in Plane Plastic Strain," by M. A. Sadowsky, *Journal of Applied Mechanics*, Vol. 8, No. 2, June, 1941, pp. A-74–A-76.

couraging further study, because they are not parallel and perpendicular to the embankment surface. If the soil contains the slightest amount of cohesion, this orthogonal boundary condition must be satisfied since a stress circle can then be drawn on a Mohr diagram passing through the origin and tangent to

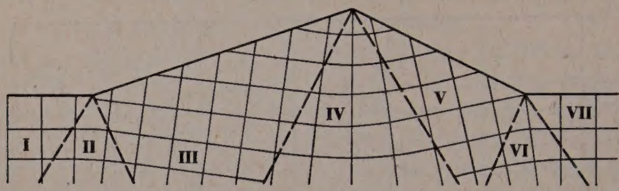

FIG. 8

the limiting line. It is possible that for a granular cohesionless mass the failure law should be represented by a stair-stepped line passing through the origin, but with extremely small steps. If such were true, a tiny circle could then be drawn tangent to the limiting line and passing through the origin of the Mohr axes. This would demand the aforementioned orthogonal boundary condition.

In an attempt to sketch a set of principal stress lines orthogonal to the boundary one may try a set of circular and radial type lines. If an attempt is made to picture the changing shape of these lines as the height of the embankment is increased from zero to a maximum, this set of lines is rejected for a set of the type sketched in Fig. 9. These lines correspond to a non-linear solution. A few interesting observations can be made from this set of principal stress lines, assuming that the embankment has the maximum possible height. At point A, Fig. 9, the principal stress σ_1 will be the minimum principal stress,

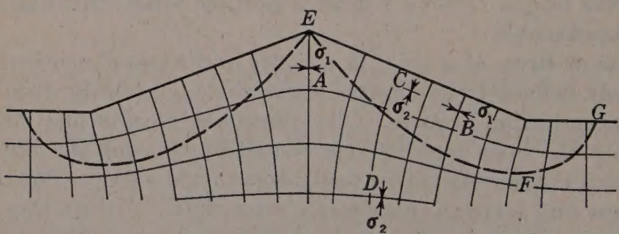

FIG. 9

whereas at point B the stress σ_1 is the maximum principal stress. Thus some point between A and B, on any path that may be chosen, is an isotropic point or one at which $\sigma_1 = \sigma_2$. At this point, Mohr's circle degenerates to a point and there are no shearing stresses in any direction. Similarly, at point C, σ_2 is the minimum principal stress and becomes the maximum at point D. The trajectory of isotropic points might appear as curve EFG. This curve would probably appear first around the fillets and expand from there to its final position as the height of the embankment is increased. Instinctively one expects the plastic state to develop around the fillets from stress concen-

tration as it occurs in elastic bodies. Heavy thrusts at the toe and heel, parallel to the surface, are in agreement with M. Frontard's solution.[8] The trajectory of isotropic points should lie within a band of elastic-state stress although the stress in such a band may be a matter of statistical mechanics rather than elasticity.

If the author's solution were applied to a symmetrical embankment constructed to maximum possible height, it appears that the principal stress lines would be vertical and horizontal. This must be questioned. This writer believes that at maximum height of embankment there will be a maximum amount of curvature and rotation of principal stress lines, which is another way to express "arching" action. The authors find Rankine's state of stress in regions I and VII of Example 1, whereas the truth would be a natural state with "at rest" pressures. This is a minor criticism. It is not apparent how the elastic and plastic regions extend downward into the foundation below the region shown in Fig. 4. The linear solution presented seems to be reasonable at an external corner, or apex of an embankment, but must be questioned when applied to a reentrant corner or fillet since it does not appear to predict the expected stress concentration.

The comments presented in this discussion are not intended to be critical of the solution presented by the authors, but rather they are meant to assist in stimulating increased interest in dealing with soil-stress problems by mathematical theory. Attention might be called to the fact that, at present, soil analysts are unable to detect experimentally the state of stress in a granular mass at various points. Hence there is no manner of proving by the "scientific method" that a given solution is true. So long as investigators are content to struggle with metallic pressure cells they will make little progress. There is great need for creative thought in both the experimental and theoretical solution of problems in the stress analysis of soils.

[8] "Calculs de Stabilité des Barrages en Terre," by M. Frontard, *Transactions*, Second Congress on Large Dams, Vol. IV, pp. 243–293.

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DISCUSSIONS

DESIGN AND CONSTRUCTION OF SAN GABRIEL DAM NO. 1

Discussion

BY WALTER L. HUBER, M. AM. SOC. C. E.

WALTER L. HUBER,[8] M. AM. SOC. C. E.[8a]—As stated by the author, a revised design for San Gabriel Dam No. 1 was approved by the state engineer of California on August 12, 1935. This approval was secured only after many conferences among the state's Board of Consulting Engineers for this dam (F. C. Herrmann, M. Am. Soc. C. E., the writer, and the late Charles D. Marx, Past-President and Hon. M. Am. Soc. C. E.), the chief engineer of the Los Angeles Flood Control District, and the consulting engineers of the District. The section of the dam, although considered conservative, was the minimum which, under all of the local conditions, was satisfactory to the state's consultants.

The capacity of spillway required by the state's consultants was 80,000 cu ft per sec with 15-ft freeboard. Had certain information since secured (partly the record of subsequent floods) been available, it is probable that a larger discharge capacity would have been required. However, the requirement of freeboard that was fixed as a condition is quite conservative, and by encroaching upon this freeboard a discharge that is ample is indicated—this without jeopardy. The model tests referred to by the author lead him to conclude "that at least 290,000 cu ft per sec would be discharged through the spillway before the dam were overtopped." As he states, this is "about three times the peak flow into the reservoir during the flood of March 2, 1938, and more than six times the peak flow on record prior to that date" (see heading "The Spillway").

NOTE.—This paper by Paul Baumann, M. Am. Soc. C. E., was published in September, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1941, by E. Soucek, Jun. Am. Soc. C. E.; December, 1941, by William P. Creager, M. Am. Soc. C. E.; January, 1942, by Joseph Jacobs, M. Am. Soc. C. E.; and February, 1942, by Jacob Feld, M. Am. Soc. C. E.

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[8a] Received by the Secretary February 9, 1942.